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INPUT-OUTPUT MODELING
AND
RESOURCE USE PROJECTION

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Input-Output Modeling

Input-output models have been found to be useful in analyzing the economic relationship or linkages among major sectors of an economy, either national or regional. In agricultural economics, we are frequently interested in using input-output models to examine the economic interrelationship between agricultural sectors such as crops and livestock and other sections of the economy such as manufacturing, services and retail trade.

As agricultural producers market their products, some are sold directly to final consumers, but most is sold to intermediate processors and/or handlers. These firms may process the raw products, combine agricultural commodities with other products and then see either to final consumers or other intermediate producers. Consider, for example, the case of cotton production and marketing. Final consumers have little use for cotton in its original form. Before it is finally consumed as shirts, pants, dresses, mattresses or some other form, the cotton passes through numerous links in the marketing and manufacturing chain. Even though farmers do not sell their cotton directly to consumers, it is clear that any change in the demand for finished cotton products will be transmitted to output requirements at the farm through the backward linkage of this chain. Using input-output analysis, it is possible to project output requirements that must be met by the agricultural sectors given a change in output in any other sector of the economy. As will be seen, many other uses also may be made of input-output analysis.

6.1 History and Evolution of Input-Output Models

The history of input-output models may be traced to the original work of Professor Wassily W. Leontief of Harvard University. Professor Leontief was interested in identifying the industrial interdependence within the American Economy and developing a
mathematical model within which all economic linkages could be stated and estimated statistically. Since this original work, the input-output technique has become the most popular interindustry model around the world.

In the U.S., the Bureau of Economic Analysis (BEA) of the U.S. Department of Commerce has a policy of completing an input-output model of the U.S. economy every five years. The most recent model completed was for each state and many regions within states. Through Regional Economic Information System (REIS) and other sources, the Department of Commerce provides upon request income and employment multipliers for counties throughout the U.S. Their fee based service can also provide multipliers for a designated group of counties as a whole.

Another source of input-output modeling, is the commercial group called the Minnesota IMPLAN Group, Incorporated or MIG-Inc. at Stillwater, Minnesota. This organization maintains and sells a national data base and input-output methodology (software) known as IMPLAN (Input-output Model for Planning). Current information is available by state and counties within states from which the user can construct input-output models for any region of choice that is at least as large as a single county. Local regional models are developed by IMPLAN software through a series of adjustments made to the national input-output model using local economic information. MIG-Inc. also provides user educational programs, seminars and workshops.

IMPLAN is the most widely used methodology available for input-output modeling. Its popularity is due to its flexibility and the extensive economic information that may be obtained through its application. Prior to IMPLAN many state governments
built and maintained state level input-output models, a tedious and expensive undertaking. Since IMPLAN most have adopted it because of its relative ease of use and low cost.

In the following sections, the basic components of the input-output model are reviewed along with its underlying assumptions. Then some examples of its usefulness in analyzing the interrelationships within an economy are examined.

6.2 Structure of an Input-Output Model

An input-output model consists of three basic tables; the transaction or flow table, a table of direct (or technical) coefficients and a table of interdependence coefficients. The transaction table is the basic data table of the model and the direct and interdependence coefficients tables are analytical matrices derived from the transaction table.

6.2.1 The Transaction Table

As its name implies, the transaction table is constructed in such a way as to identify the transactions that occur among major sectors of an economy. Each producing sector within the economy has a certain amount of output which may be used within the sector, sold as inputs to other producing sectors or sold for final demand to consumers. The transaction table summaries the annual dollar value of these sales. Table 6.1 shows a transaction table for an economy that has been subdivided into three producing sectors, agriculture, manufacturing, and services, one final demand sector, and one final payments sector. This table describes the existing structure of the economy in terms of sales and purchases of goods and services of each sector to (or from) all other sectors.

<table>
<thead>
<tr>
<th>To</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>From</td>
<td>Agriculture</td>
<td>Manufacturing</td>
<td>Services</td>
<td>Final Demand</td>
<td>Total Output</td>
</tr>
</tbody>
</table>
The producing sectors (agriculture, manufacturing and services) include all firms and industries in the economy that would normally be classified under these headings. Most models adhere closely to the U.S. Department of Commerce classification system called the Standard Industrial Classification (SIC) system. Agriculture includes livestock, crops, poultry, etc. while manufacturing includes agricultural processing, petroleum refining, steel production and all other manufacturing industries. Services include transportation, wholesale, retail as well as other business and personal services. In the example economy depicted in Table 6.1, agriculture’s total output is valued at $3290 million, while manufacturing and services outputs values are $5150 and $5270 million, respectively.

The final demand sector shown in column 4 of Table 6.1 includes the value of goods and services used by households, government, and exports to other regions. The final demand sector is known as an exogenous sector because changes in demand for products in this sector occur autonomously and its repercussions are transmitted through the rest of the economy. Changes in the final demand sector occur because of political

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Agriculture</td>
<td>390</td>
<td>1400</td>
<td>0</td>
<td>1,500</td>
<td>3290</td>
</tr>
<tr>
<td>(2) Manufacturing</td>
<td>150</td>
<td>920</td>
<td>630</td>
<td>3,450</td>
<td>5150</td>
</tr>
<tr>
<td>(3) Services</td>
<td>240</td>
<td>860</td>
<td>1520</td>
<td>2,650</td>
<td>5270</td>
</tr>
<tr>
<td>----------------</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
</tr>
<tr>
<td>(4) Final Payments</td>
<td>3290</td>
<td>1970</td>
<td>3120</td>
<td>6,110</td>
<td>13710</td>
</tr>
<tr>
<td>(5) Total Outlay</td>
<td>3290</td>
<td>5150</td>
<td>5270</td>
<td>13710</td>
<td></td>
</tr>
</tbody>
</table>
decisions and consumer preferences. Tracing the direct and indirect effects of a change in
the exogenous, final demand sector on the producing (endogenous) sectors is the primary
objective of an input-output model.

The final payment sectors account for the direct payments for such items as wages,
salaries, other labor income, proprietor income, including profits and payments made
outside the region for goods and services imported. Final payments, such as imports, are
considered to be leakages from the local economy, whether it be a county, state or region
since money paid does not reenter the local economic structure.

All input-output transaction tables must be balanced in that the total output and
each producing sector must be equal to its total outlay as indicated in row 5 of column 5
of Table 6.1. This is an accounting requirement so that no economic activity is lost (or
gained) in the organization of the transaction table and all income and outlay is accounted
for.

The row entries in a transaction table describe the way in which the total sales or
each sector are allocated over the remaining sectors in the economy. For example, the
manufacturing sector (row 2, Table 6.1) sells $150 million worth of goods and services to
agriculture, $920 million to manufacturing (to other manufacturing firms), $630 million to
the service sectors and $3,450 million to the final demand sector.

The column entries in the table describe the inputs or purchases side of each sector
in relation to all other sectors. Again considering the manufacturing sector, column 2 or
Table 6.1 show that $1,400 million worth of products were purchased by manufacturing
from agriculture, $920 million from manufacturing internally and $860 million from
services. Also $1,970 million was paid to labor, taxes, depreciation, imports and other
items by the manufacturing sector in the production of its total output as indicated by the final payments entry. When all purchases or expenditures by sector are considered, total sector output is exactly equal to total sector outlay.

In most input-output models, the transaction table would be greatly expanded and each of the sectors shown in Table 6.1 would be disaggregated into several component sectors. Nevertheless, the interpretation of the entries in the transaction table is the same regardless of the level of disaggregation used in the input-output analysis.

6.2.2 Technical Coefficients

While the transaction table provides an interesting and useful “snapshot” of the structure of an economy, it is only descriptive of the current situation is not very useful for economic analysis. To use input-output analytically to examine how production in each sector will change in response to a change in the demand for final product, we must first derive the technical coefficients.

Technical coefficients show the production function for each processing sector and therefore they are not relevant for the final demand sector. Technical coefficients show the value of inputs purchased from all sectors in the economy per dollar of output in a particular sector. They are based on three simplifying, but important assumptions.
These are:

1. Each sector produces only one homogeneous commodity.
2. Each sector has a fixed input-output ratio.
3. Each sector operates under conditions of constant returns to scale.

Using these assumptions and the example economy shown in Table 6.1, technical coefficients may be derived by dividing all entries in each sector’s column by the total outlay of that sector shown in Table 6.2.

**Table 6.2 Example of Technical Coefficients in an Input-Output Model.**

<table>
<thead>
<tr>
<th>Sector</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture</td>
<td>.118</td>
<td>.272</td>
<td>0</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>.045</td>
<td>.179</td>
<td>.119</td>
</tr>
<tr>
<td>Services</td>
<td>.073</td>
<td>.167</td>
<td>.288</td>
</tr>
</tbody>
</table>

If from the transaction table, we let $x_{ij}$ symbolize the value of sales from sector I to sector j and $x_i$ the total output of sector j, the technical coefficients for each sector are calculated using the following equation:

$$a_{ij} = \frac{x_{ij}}{x_j} \quad (6.1)$$

Hence, for the agricultural sector (sector 1), the technical coefficients are calculated to be:

- Agriculture, $a_{11} = \frac{390}{3290} = .118$
- Manufacturing, $a_{21} = \frac{150}{3290} = .045$
- Services, $a_{31} = \frac{240}{3290} = .073$
These coefficients make up the first column of Table 6.2. Technical coefficients for sectors 2 and 3 are calculated in the same manner, following equation 6.1. The agricultural technical coefficients show the value of purchases from each sector in the economy that must be made by the agricultural sector in order for it to produce one dollar’s worth of output. That is, for each dollar of output produced by the agricultural sector, it must purchase $.118 (11.8 cents) from within the agricultural sector, $.045 from manufacturing and $.073 from services. Likewise, for each dollar of output by manufacturing, $.272 are purchased from agriculture, $.179 from manufacturing and $.167 from services. Technical coefficients of the services sector indicate that for each dollar of output nothing is bought from agriculture, but $.119 and $.288 are purchased from manufacturing and services, respectively. These coefficients show the direct effects in all sectors due to a one dollar change in output in a particular sector and in doing so they reveal the interindustry linkages that tie the economy together and the first round effects of an economic change such as an increase in exports. However, as we know from economic theory, direct or first round effects measure only a fraction of the total economic impacts since there are also indirect effects from any economic change.

Consider a numerical example in which we assume that exports of agricultural products increase by one dollar. What will be the total impact of this change in terms of increased output by all sectors of the economy depicted in Tables 6.1 and 6.2. We can estimate a part of this total impact by tracing through the first and second rounds of output changes caused by the change in demand. To do this, refer to Table 6.2. If final demand of the agricultural sector increase by $1.00 and intraindustry output increased by $.118 (column 1 and row 1 of Table 6.2) then output of the agricultural sector increases
by $1.118. Also, as indicated in column 1 of Table 6.2, manufacturing increases output by 
$.045 and services increases output by $.073 in order to meet the increased demand of 
$1.00. hence, through the first round of effects total output in the economy is increased 
by $1.236 (1.118 + .045 + .073) as a result of the $1.00 increase in agricultural final 
demand.

The economic repercussions of the change in agricultural final demand do not end with 
the first round effects. Because of output increases in the first round, each of these 
sectors will experience a need to increase output in the second round and then in a third 
round, and so forth. According to economic theory, the total output increases required to 
meet the change in final demand of $1.00 will include the sum of an infinite number of 
successively smaller and smaller indirect output effects.

To demonstrate this, we may examine the effects of agriculture’s final demand 
increase through the second round. It was found that, in the first round, output increases 
were required as follows: agriculture: = $.118 (not including the $1.00 final demand), 
manufacturing = $.045 and services = $.073. Second round total output requirements are 
found by multiplying each of these sector’s first round output requirements by its 
respective column elements and summing the products. Hence, in the second round the 
total dollar value of output requirements will be as follows:
First Round Column Coefficients (Table 6.2)

Output Increase

$.118 \quad (.118 + .045 + .073) = .028
$.045 \quad (.045 + .179 + .167) = .027
$.073 \quad (0 + .119 + .288) = .085

Total second round output increase = .085

In the second round, the sum of all sector’s required to meet the needs of the first round production is $.085. Hence, through the second round the total output of the economy stimulated by the original increase in agricultural final demand of $1.00 is the sum of the first and second round effects, a total of $1.321 ($1.236 from the first round plus $.085 from the second round). Now, these second round increases give rise to further output increases in the third round, fourth round and so forth, ad infinitum.

Fortunately, it is not necessary to trace through all successive rounds of output changes to find the sum of all output requirements. The total (direct and indirect) output levels needed to just satisfy specified levels of final demand may be found by deriving the interdependence coefficients matrix following the methods developed by Leontief.

6.2.3 The Interdependence Coefficients Matrix

The interdependence coefficients matrix is the most important of three input-output matrices for economic analysis purposes. The coefficients or elements of this matrix measure the total (direct and indirect) output required of all sectors in order for any particular sector to make a sale of one dollar to final demand. In other words, it measures the total impact of a change in final demand in a given sector on the output of all other
sectors of the economy after all successive rounds of output increases have been recorded. As we find in this section, these totals may be found by expressing the transaction table as a set of simultaneous equations and solving the set by means of matrix algebra. The transactions of the producing sectors of Table 6.1 may be written as a set of simultaneous equations as follows:

\[
\begin{align*}
  x_{11} + x_{12} + x_{13} + Y_1 &= X_1 \\
  x_{21} + x_{22} + x_{23} + Y_2 &= X_2 \\
  x_{31} + x_{32} + x_{33} + Y_3 &= X_3
\end{align*}
\]  

(6.2)

where,

\(x_{ij}\) = sales from sector I (rows) to sector j (column)

\(Y_i\) = sales from sector I to final demand

\(X_i\) = total output of sector I

Equation 6.2 represents the three sector economy also depicted in Table 6.1. In equation 6.1, we defined the interindustry relationships among sectors as \(a_{ij} = x_{ij}/X_j\). This expression may be rearranged to read \(x_{ij} = a_{ij} \cdot X_j\) which is interpreted to mean that the level of sales from sector I to sector j depends upon the level of output in sector j (\(X_j\)) and the technical coefficient of input requirements of sector j from sector I (\(a_{11}\)).
Substituting equation 6.1 into equation 6.2, we rewrite the equations for the producing sectors (i = 1, ..., 3) as,

\[ a_{11}X_1 + a_{12}X_2 + a_{13}X_3 + Y_1 = X_1 \]
\[ a_{21}X_1 + a_{22}X_2 + a_{23}X_3 + Y_2 = X_2 \]
\[ a_{31}X_1 + a_{32}X_2 + a_{33}X_3 + Y_3 = X_3 \]

Equation 6.3 reveals the interdependence of each sector on all others because it indicates that the level of output in any sector is dependent upon the level of output in other sectors, the input requirements of each sector and the level of its final demand. We treat final demand \(Y_i\) as exogenous to the sectors of our written:

\[ X_1 - a_{11}X_1 - a_{12}X_2 - a_{13}X_3 = Y_1 \]
\[ -a_{21}X_2 + X_2 - a_{22}X_2 - a_{23}X_3 = Y_2 \]
\[ -a_{31}X_1 + a_{32}X_2 + X_3 - a_{33}X_3 = Y_3 \]

or,

\[ (1 - a_{11})X_1 - a_{12}X_2 - a_{13}X_3 = Y_1 \]
\[ -a_{21}X_1 + (1 - a_{22})X_2 - a_{23}X_3 = Y_2 \]
\[ -a_{31}X_1 - a_{32}X_2 + (1 - a_{33})X_3 = Y_3 \]

This system can be then simplified by presenting it in matrix notation as,

\[
\begin{bmatrix}
(1 - a_{11}) & -a_{12} & -a_{13} \\
-a_{21} & (1 - a_{22}) & -a_{23} \\
-a_{31} & -a_{32} & (1 - a_{33})
\end{bmatrix}
\begin{bmatrix}
X_1 \\
X_2 \\
X_3
\end{bmatrix}
= 
\begin{bmatrix}
Y_1 \\
Y_2 \\
Y_3
\end{bmatrix}
\]

or simply \(AX = Y\). (6.5)
The $a_{ij}$ elements of the first matrix ($A^*$) of equation 6.5 have the same values as the technical coefficients as calculated from equation 6.1. Observe that except for the diagonal elements of matrix $A^*$, the only difference in the $a_{ij}$ elements from those in equation 6.1 is that they are now preceded by a minus sign. The diagonal elements are all subtracted from one. Hence, the matrix $A^*$ is the difference between two matrices - an identify matrix ($I$) minus the matrix of technical coefficients. Therefore, equations 6.5 may be written as,

$$(I - A)X = Y$$

(6.6)

where, $(I - A) = A^*$.

The solution that expresses each sector output $(X)$ as a function of final demand $(Y)$ may be found by the following manipulation. First premultiply each side of equation (6.6) by $(I-A)^{-1}$ to yield

$$(I-A)^{-1}(I-A)X = (I-A)^{-1}Y.$$  

Recalling the rules governing identity matrices, equation 6.7 becomes,

$$IX = (I-A)^{-1}Y$$

or,

$$X = (I-A)^{-1}Y$$

(6.8)

Equation 6.8 is the solution equation to the input-output system. Using this equation, we may find the levels of output from all sectors required to support specified levels of final demands in all sectors. The $(I-A)^{-1}$ matrix is the key matrix in the equation. The elements of this matrix measure the direct and indirect output levels from each sector of the economy required to satisfy given levels of final demand. It is called the matrix of interdependence coefficients.
Now let’s turn to deriving the matrix interdependence coefficients from our three sector economy that includes agriculture, manufacturing and services sectors. The matrix of technical coefficients \( A \) was shown in Table 6.2 to be:

\[
A = \begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{bmatrix} = \begin{bmatrix}
.118 & .272 & 0 \\
.045 & .179 & .119 \\
.073 & .167 & .288
\end{bmatrix}
\]

(6.9)

Thus, the difference matrix \((I-A)\) from equation 6.8 becomes,

\[
(I-A) = \begin{bmatrix}
.88 & -.27 & 0 \\
-.04 & .82 & -.12 \\
-.07 & -.17 & .71
\end{bmatrix}
\]

(6.10)

To complete our input-output equation, we invert this difference matrix to find \((I-A)^{-1}\). Following the matrix inversion from Chapter 5, the determinant of \((I-A)\) is steps in the cofactor method of found to be 2.055. The adjoint of \((I-A)\) is

\[
adj.(I-A) = \begin{bmatrix}
2.386 & .818 & .137 \\
.174 & 2.651 & .445 \\
.285 & .706 & 3.007
\end{bmatrix}
\]

(6.11)
Then, dividing each element of the adjoint matrix by the determinant we find that the matrix of interdependence coefficients is,

\[
(I-A)^{-1} = \begin{bmatrix}
1.161 & .398 & .067 \\
.085 & 1.289 & .217 \\
.138 & .343 & 1.463 \\
\end{bmatrix}
\]  

(6.12)

Before using this matrix to estimate the total sector output levels in our example, let’s look at the interpretation of its elements. Each column of the \((I-A)^{-1}\) matrix corresponds in the same order to the original economic sectors shown in Table 6.1. That is, column 1 contains the coefficient for the agricultural sector, column 2 contains those for manufacturing and column 3 those for services. Consider column 2 for purposes of interpretation. The coefficients of column 2 of the matrix indicate that for each dollar of sales to final demand by the manufacturing sector, total output requirements are $.398 from agriculture, $1.289 from manufacturing and $.343 from services. The output required from manufacturing includes its $1.00 sales going to final demand, and $.289 of additional indirect output that is brought about by the fact that other processes and that they must increase output to satisfy the increase in final demand experience by manufacturing.

As was shown in Table 6.2 and equation 6.8, the direct input requirements of manufacturing from agriculture are $.27 per $1.00 of output. However, the total output requirements from agriculture is $.398 for each $1.00 sales by manufacturing to final demand as shown in equation 6.12. The difference between the total effect and direct effect ($0.398 - $0.27 = $0.128) is the indirect output required from agriculture. Not all of this indirect output will be sold to manufacturing as is the case with the direct output.
Rather, $.128 of output is required of the agricultural sector to meet the indirect input needs of all sectors of the economy.

6.4 Projection of Output

Now, having estimated the interdependence coefficients, we can use the results shown in equation 6.12 for several analytical purposes. The projection of total output required to satisfy new levels of final demand is one of the frequent uses of input-output models.

The levels of total output shown in column 5 of Table 6.1 correspond to the final demand levels shown in column 4 of Table 6.1. This may be verified by substituting the final demand values (agriculture = 1500, manufacturing = 3450 and services = 2650) into the basic input-output equation and premultiplying the interdependence coefficients matrix by the sector of final demands. Writing out the complete matrices, we get the following empirical equation and results,

$$\begin{bmatrix}
X_1 \\
X_2 \\
X_3
\end{bmatrix} = 
\begin{bmatrix}
1.161 & .398 & .067 \\
.085 & 1.298 & .217 \\
.138 & .343 & 1.463
\end{bmatrix} \cdot 
\begin{bmatrix}
1500 \\
3450 \\
2650
\end{bmatrix} = 
\begin{bmatrix}
3290 \\
5150 \\
5270
\end{bmatrix} \quad (6.13)$$

Equation 6.13 reveals that so long as final demands of each sector remain at the levels shown, total output requirements will also remain at the indicated levels. Therefore, if exports, consumer demand, and other components of final demand are projected for the next time period (say next year) to remain unchanged for all sectors, the projected level of economic activity will also be the same as shown in equation 6.13.
However, if final demand is expected to increase next year, then new and higher levels of sector outputs will be required. One important feature of input-output analysis is that so long as the input requirements among the sectors remain the same, the \((I-A)^{-1}\) coefficients will not change. Therefore, only one matrix inversion is required even if we consider numerous different levels of final demand. This means that sector outputs required to satisfy next year’s final demand levels may be projected by substituting new final demand values into equation 6.13 and solving for new sector output levels.

Let’s assume, for example that for the next year manufacturing and services final demand are not expected to change, but that agriculture is expected to experience an increase of $50 million in exports so that its final demand goes up from $1500 to $1550 million. Hence, the expected final demand levels for next year may be written in the final demand vector as follows:

\[
Y_{t+1} = \begin{bmatrix} 1550 \\ 3450 \\ 2650 \end{bmatrix}
\]

(6.14)

Next year’s sector output levels are found by substituting \(Y_{t+1}\) in equation 6.14 for \(Y\) in equation 6.13 and solving the equation \(X_{t+1} = (I-A)^{-1} \cdot Y_{t+1}\) to get the following results:

\[
\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 1.161 & .398 & .067 \\ .085 & 1.298 & .217 \\ .138 & .343 & 1.463 \end{bmatrix} \cdot \begin{bmatrix} 1550 \\ 3450 \\ 2650 \end{bmatrix} = \begin{bmatrix} 3348 \\ 5154 \\ 5277 \end{bmatrix}
\]

(6.15)

A comparison of sector output levels in equation 6.15 with those in equation 6.13 reveals that all sectors must increase output to meet the requirements created by the $50 million increase in agricultural exports. Agricultural sector output must increase from
$3290 to $3348 million, manufacturing from $5150 to $5154 million and services from $5270 to $5277 million. The changes in output requirements for the agricultural sector ($3448 - 3290 = $58 million) is due to both the final demand change ($50 million) and the indirect output requirement of other sectors ($8 million). Neither the manufacturing sector nor the services sector experienced an increase in final demand. Nevertheless, the input-output requirements increased by $4 million and $7 million, respectively, reflecting the indirect effects of the sector interdependence captured by the input-output model. In this case, we considered a change in agriculture’s final demand only. We can just as easily estimate the effects on output requirements of a change in some other sector’s final demand - either in isolation or in any combination. And, as stated earlier, so long as input requirements among sectors do not change, we can examine as many cases as we wish to simply by altering final demand levels and performing a matrix multiplication as demonstrated in equation 6.15.

6.5 The Final Demand Multiplier

One of the most widely used concepts in economics is that of the multiplier. A frequent question asked by agricultural associations, industrial groups and other is, “what overall impact does my industry have on the economy?” Stated differently, the question of interest is what total effect will change in the sales of a given sector of the economy have on total output of that economy. The answer to this question can be obtained from input-output analysis using an estimate called the final demand multiplier. Each sector has a unique multiplier that expresses that total change in output from all sectors of the economy for each one dollar change in final demand in that sector. In the example
considered in the previous section we examined the effect of a change in agriculture’s final demand of $50 million on all sectors of the economy. The result of that change was estimated to be:

Sector 1, agriculture - $58 million
Sector 2, manufacturing - $4 million
Sector 3, services - $7 million
Total of all sectors - $69 million

The $50 million change in agricultural final demand has an impact of $69 million on total output of the economy. Hence, the final demand multiplier (the ratio of total output change to final demand change) may be found as follows:

\[
\text{Final Demand Multiplier} = \frac{\text{change in total output}}{\text{change in final demand}} = \frac{69 \text{ million}}{50 \text{ million}} = 1.38
\]

Similar calculations for the manufacturing and services sectors would produce final demand multipliers for those sectors. However, such round-about calculations are not necessary since the final demand multipliers can be found directly from the \((I-A)^{-1}\) matrix. To do this, consider a final demand vector that consist of one unit of agricultural output and nothing else such that the elements of that vector are,

\[
Y = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}
\]

Let \(c_{ij}\)'s be the elements of the interdependence coefficients, \((I-A)^{-1}\). Then total output requirements needed to satisfy the final demand in agricultural may be calculated as,
Therefore, the first column of $(I-A)^{-1}$ matrix indicates the total output required from each sector to produce one unit of final demand for sector one. Since each element in the column shows that total output required of each sector per dollar of final demand of that column sector, it follows that if we add up the elements of each column, we will get the total output required of the entire economy. Given our example matrix (equation 6.12), the sum of the elements in each column yields the following final demand multipliers,

- Sector 1, agriculture = $1.61 + 0.085 + 0.138 = 1.834$
- Sector 2, manufacturing = $0.398 + 1.298 + 0.343 = 2.039$
- Sector 3, services = $0.067 + 0.217 + 1.463 = 1.747$

Each of these multipliers indicates the total change in output in the economy required per dollar of final demand a particular sector. As it is frequently interpreted, we can say that if the manufacturing sector increases (decreases) its sales to final demand by one dollar, total output of the economy will increase (decrease) by $2,039. So, these multipliers may be used to estimate the economic impact of changes in specific sectors on the general economy.

6.6 Input Requirements of Primary Resources

When output increases to meet final demand, requirements for primary resources such as labor, land, water, etc. also increase. The direct resource requirements per dollar
of output may be expressed as the ratio of total resources use by a sector to total sector output. But this direct resource requirement is not enough to measure the total impact on resource use in the economy since it ignores the indirect resource requirements. The interdependence coefficients matrix may be used to translate gross output changes into forecasts of aggregate resource requirements. Since resources are limited, this provides an added, important use of the input-output model. We are interested in projecting total resource requirements to find out if the total required is consistent with what is available in the economy.

If the functional relationship between resource requirements and output is specified, total resource requirements may be computed for a specified final demand. The relationship between resource requirement and output may be stated as,

\[ r_{kj} = b_{kj}X_j \]

where,

- \( r_{kj} \) = the amount of primary resource \( k \) required by sector \( j \)
- \( X_j \) = output of sector \( j \)
- \( b_{kj} \) = the amount of resource \( k \) used per dollar of output in sector \( j \)

Thus, the technical resource coefficients \( b_{kj} \) are derived from a single observation of the resource requirement for sector \( j \) by the ratio,

\[ b_{kj} = \frac{r_{kj}}{X_j} \]

Let \( R_k \) equal the total requirement for resource \( k \). Then,
\[ R_k = \sum_{j=1}^{n} r_{kj} = \sum_{j=1}^{n} b_{kj} X_j \]

\[ = b_{kj_1} X_1 + b_{kj_2} X_2 + \ldots + b_{kn} X_n \text{ for } k=1,2,\ldots,m \text{ resources.} \]

Expanding this equation, the demand for resources 1 through m is

\[ R_1 = b_{11} X_1 + b_{12} X_2 + \ldots + b_{1n} X_n \]
\[ R_2 = b_{21} X_1 + b_{22} X_2 + \ldots + b_{2n} X_n \]
\[ R_m = b_{m1} X_1 + b_{m2} X_2 + \ldots + b_{mn} X_n \]

In matrix notation,

\[ R = BX \quad (6.16) \]

where,

\[ R = \begin{bmatrix} R_1 \\ R_2 \\ \vdots \\ R_m \end{bmatrix}, \quad B = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{bmatrix}, \quad X = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix} \]

It has already been established in equation 6.8 that,
\[ X = (I-A)^{-1}Y \]

Substituting equation 6.8 into equation 6.16 the total demand for resources is,

\[ R = B(I-A)^{-1}Y \quad (6.17) \]

or,

\[ R = PY, \text{ where } P = B(I-A)^{-1} \]

The matrix \((P)\) shows the direct and indirect effect of a final demand change on the various resources used in production. Let \(p_{kj}\) be an element in the matrix \(P\). Then, for every dollar’s worth of final demand for products of sector \(j\), \(p_{kj}\) units of resource \(k\) are required directly by producing sector \(j\) and indirectly through requirements induced by output changes in other sectors of the economy. The matrix of resource requirements \(®\) is important for planning purposes since they set the capacity constraints within which economic change may occur.

Consider our earlier example from section 6.4 in which next year’s total output for each sector was projected to be,

Sector 1, agriculture = 3348
Sector 2, manufacturing = 5154
Sector 3, services = 5277

The projected outputs are each sector’s requirement needed to meet both the direct sales to final demand and the secondary outputs caused by the economic interactions. Now, say the question arises as to whether or not a sufficient amount of some basic resource - say labor, is available to allow these levels of output to be reached. Let the direct labor requirements per dollar of output \((b_{kj})\) for each sector be: Sector 1, agriculture = 60 workers per million dollars of output; Sector 2, manufacturing = 35
workers per million dollars of output; Sector 3, services = 52 workers per million dollars of output.

Given these labor to output ratios (in this case expanded to workers per million dollars of output) we may use either equation 6.16 or 6.17 to solve for the total labor required to meet our specified level of final demand. Using equation 6.16, we find that labor required will be, \((60 \times 3348) + (35 \times 5154) + (52 \times 5277) = 655,674\) workers.

Therefore, the projected levels of output of each sector can be met for the next year only if 655,674 workers are available in the economy. In regional applications this may indicate the number of workers who will need to migrate into the region to meet labor requirements if the local labor force is insufficient to meet the demand.

### 6.6.1 An Employment/Final Demand Multiplier

Before the projected increase in agriculture final demand of $50 million, total employment in the economy was 651,690 jobs (Table 6.1). The increase in sales to final demand by agriculture stimulated direct and indirect employment requirements of 3,984 jobs \((655,674 - 651,690)\). Given this information, we can derive the employment to final demand multiplier by dividing the change in employment by the change in final demand, \((3984/50=79.68)\). This multiplier tells us the projected change in employment per dollar change in sales to final demand by agriculture, other things equal. Similar multipliers may be derived for any resource or economic variable that may be expressed as linear function of sector output as shown in equation 6.16.

Caution should be emphasized in closing that models such input-output are designed for a specific purpose and are limited by the underlying assumptions upon which
they are built. In fact, the assumptions inherent in input-output analysis are strict and quite limiting. Because of these assumptions, input-output coefficients and results do not yield marginal effects. For example, the employment to output ratio used in this section is an average ratio and does not necessarily show the marginal increase in labor required to expand final demand by the given amount. This would only be the case under conditions of full employment and fixed technology. Input use probably does respond to output increases proportionally as is assumed in input-output analysis.

A second caution has to do with the underlying purpose and use of input-output analysis. It is a positivistic model intended for planning purposes such as projecting changes in job requirements, resource requirements and industrial output requirements resulting from stimulus in the economy. As such, it is used widely and has proven to be a very beneficial planning tool. However, this model was never intended to be a valuation tool that could be used in determining the value of or benefits from the project. In this regard, the model has been misused a great deal. It is not and should not be used as a benefit-cost assessment model. The theory and practice of benefit-cost analysis is well developed and documented. Input-output analysis is not a benefit-cost analysis model. A frequent example is that of a public project to be developed with, say, state funds and located in a community with expectation of stimulating economic growth. Supporters of the project may wish to look to direct and indirect jobs created and income increases as benefits to be used in justifying the project. However, from the perspective of the state, this is only one of many projects that could be initiated. The other projects would also stimulate direct and indirect economic growth. So, to count the indirect effects of a particular project over states it benefits vis-a'-vis other potential projects. Nevertheless,
and to end this little lesson on a positive note, input output modeling is in great demand. It provides information that is difficult, if not impossible, to gain with other models. Remember that there is one correct use of the model and an infinite number of incorrect uses.