PhD Qualifier Examination

Department of Agricultural Economics

July 29, 2019

Instructions

This exam consists of **six** questions. You must answer all questions. If you need an assumption to complete a question, state the assumption clearly and proceed. Be as clear as possible in your answer. You have four hours to complete the exam. Show all your work. If necessary, use math, graphical analysis and provide definitions of key concepts.

- Be sure to put your assigned letter and no other identifying information on each page of your answer sheets.
- Also, put the question number and answer page number (e.g. 4-1) at the top of each page.
- Write on only one side of your paper and leave at least 1 inch margins on all sides.
- Make sure your writing is clear and easy to read.
- Turn in your final copy with all pages in order.

GOOD LUCK!

1. (15 points) Let $X_i, i = 1, \ldots, N$, be an I.I.D. sample from a distribution with mean μ and variance $\sigma^2 < \infty$. Consider the following two estimators of μ: $\hat{\mu} = 1/N \sum_{i=1}^{N} X_i$ and $\tilde{\mu} = 1/(N-1) \sum_{i=1}^{N} X_i$.
   (a) Derive the expectations of $\hat{\mu}$ and $\tilde{\mu}$.
   (b) Derive the variances of $\hat{\mu}$ and $\tilde{\mu}$.
   (c) Derive the mean squared errors of $\hat{\mu}$ and $\tilde{\mu}$.
   (d) Show that both $\hat{\mu}$ and $\tilde{\mu}$ are consistent estimators of μ.

2. (15 points) Consider the model $Y_t = X_t\beta + e_t, t = 1, \ldots, T$, where $e_t = \rho e_{t-1} + u_t$, $E[u_t] = 0$, $\text{Var}(u_t) = \sigma_u^2 < \infty$ and $\text{Cov}(u_t, u_s) = 0$ for $t \neq s$. Define $u = (u_1, \ldots, u_T)^T$ and $\text{Var}(u) = \Omega$.
   (a) Derive Ω in terms of σ_u^2 and ρ.
   (b) Show that the model is stationary only if $|\rho| < 1$.
   (c) Propose a consistent estimator of Ω.
   (d) Propose an asymptotically efficient estimator of β.

3. (20 points) Consider a first price, full information, all-pay auction in which it is common knowledge that all $n \geq 2$ risk neutral players value the object at $v > 0$. The bidders submit their non-negative bids $b_1, b_2, \ldots, b_n$, and the object is awarded to the individual with the highest bid, and everyone pays his bid whether he wins or loses, with ties being broken randomly without bias. Thus, the winner's *ex-post* payoff is $v - b$, and a loser's *ex-post* payoff is $-b$.
   (a) Prove that there is no symmetric pure strategy Nash equilibrium of this auction game.

   Now suppose we are looking for a symmetric mixed strategy Nash equilibrium (SMNE). Let the mixed strategy equilibrium be represented by a cdf $F(b)$.

   (b) Suppose player 1 bids b. Give an expression for his probability of winning the object if all other players play the mixed strategy $F(\cdot)$? Assume that players' bids are independent.
   (c) What is player 1's expected payoff of submitting the bid b?
   (d) Derive the equilibrium cdf $F(\cdot)$ that corresponds to the SMNE.

   Now consider a slightly different auction in which everyone submits $b_i \geq 0$ and the highest bidder gets the object, but now everyone pays the *second* highest bid, *i.e.* a full information, second price, all-pay auction. As before, we are looking for a symmetric mixed Nash equilibrium (SMNE), which is represented by a cdf $F(b)$. Also, for simplicity suppose there are only two bidders.

   (e) What is the *ex-post* payoff of each bidder $u_1(b_1, b_2; v)$ and $u_2(b_1, b_2; v)$, when $b_1 < b_2$ and $b_2 < b_1$?

4. (15 points) Consider a corporate venture capital (CVC) whose business is to fund entrepreneurs to start up their projects. In return, the CVC takes a share of the output. In our simple setting, all projects cost the same for CVC, which we normalize to zero. However, entrepreneurs have different types θ_H and θ_L, with $\theta_H = 2$, and $\theta_L = 1$, and $\Pr\{\theta = \theta_H\} = \Pr\{\theta = \theta_L\} = \frac{1}{2}$. High type entrepreneurs (θ_H) have higher productivity, but the types are private knowledge,

and since the CVC can't observe the types, it decides to give contracts based on output (which is observable and contractable). Namely, a contract (s, y) means that the entrepreneur, if funded, has to produce y from which the CVC takes the share s (so the CVC's payoff from this contract is sy). A type θ entrepreneurs' payoff from the contract (s, y) is $(1-s)y - \frac{y^2}{\theta}$, and both types have reservation values of zero.

(a) Write down the CVC's program for finding an optimal mechanism $(s_H, y_H; s_L, y_L)$ to maximize her expected payoff. Include *all* the constraints)

(b) State which constraints should bind.

(c) Based on (b), write down the Lagrangian and solve for the optimal mechanism.

(d) Suppose the CVC had full information. What would be its offer (s_H, y_H) and (s_L, y_L) to different types?

(e) Is there any inefficiency (in production) resulting from the optimal mechanism (part a)? Explain.

5. (15 points) Consider a complete, transitive, continuous, and monotone binary relation $\succsim$ defined on $\mathbb{R}^L_+$. Consider the following function. For each $x \in \mathbb{R}^L_+$,

$$u(x) \equiv \min\{||y|| : y \in U(\succsim, x)\},$$

where $||y|| \equiv \left(\sum_{l=1}^{L} y_l^2\right)^{1/2}$ is the Euclidean norm of y, which is the length of vector y, and also the distance from the point y to 0; and $U(\succsim, x) \equiv \{z \in \mathbb{R}^L_+ : z \succsim x\}$.

(a) Prove that u is well-defined. That means, for each $x \in \mathbb{R}^L_+$, the problem

$$\min\{||y|| : y \in U(\succsim, x)\}, \tag{1}$$

has solution. You can use the fact that $|| \cdot ||$ is a continuous function. You need to state here the theorem that guarantees existence of a solution to a continuous maximization problem. You will find that the conditions are not satisfied here. So your challenge is to go around this problem and find another problem that you can guarantee has solution and whose solution is also a solution to (1).

(b) You will now advance in the proof that the u defined above represents $\succsim$.

1. State the definition of u represents $\succsim$. Do it in two independent conditions (not an iff condition).
   (a) Independent Condition A.
   (b) Independent Condition B.

(c) Choose one of the statements above, i.e., A or B, and prove it. (Clearly state the one you will prove).

6. (20 points) Consider an Expected Utility maximizer, on the set of money lotteries, whose utility index is:

$$u(x) \equiv -e^{-ax},$$

where $a > 0$.

(a) What function represents the preferences of the agent described above?

(b) Define what is a Risk averse agent.

(c) Is the agent described above Risk averse? (prove it if so, or disprove it if not)

(d) Imagine that this agent has an initial wealth $w > 0$. Let F be the lottery that multiplies the agent's wealth by a factor $k > 0$ with probability α and gives the agent 0 dollars with probability $(1 - \alpha)$.

1. Show that for any given $k > 1$ there is $0 < \alpha^* < 1$ such that the agent prefers lottery F to keeping her wealth for sure whenever $\alpha > \alpha^*$.
2. Now, show that for any k and any $0 < \alpha < 1$, if the agent's initial wealth is sufficiency large, the agent prefers to keep her wealth for sure instead of lottery F.