PhD Qualifier Examination

Department of Agricultural Economics

June 29, 2020

Instructions

This exam consists of **six** questions. You must answer all questions. If you need an assumption to complete a question, state the assumption clearly and proceed. Be as clear as possible in your answer. You have four hours to complete the exam. Show all your work. If necessary, use math, graphical analysis and provide definitions of key concepts.

- Be sure to put your assigned letter and no other identifying information on each page of your answer sheets.
- Also, put the question number and answer page number (e.g. 4-1) at the top of each page.
- Write on only one side of your paper and leave at least 1 inch margins on all sides.
- Make sure your writing is clear and easy to read.
- Turn in your final copy with all pages in order.

GOOD LUCK!

1. (15 points) Consider the model

$$Y_i = X_i^T \beta + e_i, i = 1, \dots, N, \tag{1}$$

where X_i is a $K \times 1$ vector of covariates, β is $K \times 1$ unknown coefficient, and e_i's are i.i.d. random errors with density function

$$f(e_i; \sigma) = \frac{1}{\sqrt{2\pi}\sigma} \exp(-\frac{e_i^2}{2\sigma^2}),$$

where $0 < \sigma < \infty$.

(a) Write down the log-likelihood function of the Maximum Likelihood Estimator (MLE) of model (1).

(b) Derive the MLE of β. Show that it is identical to the OLS estimator of β.

(c) Derive the MLE of σ^2.

2. (20 points) An analyst estimates the following specification over the period 2010.1 to 2019.4 using OLS:

$$ln(sales) = 3.95 - 322.08P + 163.15PS + 0.0173Q_1 - 0.0586Q_2 - 0.0677Q_3 \tag{2}$$

Q1, Q2, and Q3 are quarterly dummy variables to account for seasonality; P denotes own-price, and PS denotes the price of a substitute/complementary product.

In addition, suppose that the sample means of the respective variables over this time period are as follows:

SALES=55.74; P=\$0.0043/unit; and PS=\$0.0035/unit.

(a) On the basis of this information, what is the own-price elasticity of demand?

(b) Is the demand for this product elastic? Why or why not?

(c) If PS were to change by \$0.001/unit, what is the impact on SALES? Use the sample means of SALES in this calculation. Does PS refer to a substitute or a complement? Defend your answer.

(d) Characterize the level of sales in quarter 2 relative to quarter 4.

(e) Forecast the level of sales for quarter 1 when P=\$0.005/unit and when PS=\$0.004/unit.

(f) What additional explanatory variable(s) would you add, if any, to the specification of this demand model? Defend your answer.

(g) Suppose that the regression sum of squares is equal to 90, and that the error or residual sum of squares is equal to 30. What is the adjusted R2 for this model?

(h) Suppose that the Jarque-Bera test statistic associated with the residuals has a p-value of 0.75. The partial autocorrelation function for the residuals at a lag of 2 is statistically significant. All other partial correlation coefficients are not statistically different from zero. What can be said about the behavior of the residuals?

(i) From your answer in (h), what would you recommend econometrically?

(j) How would you determine the presence of degrading collinearity in this analysis? If present, what would you recommend econometrically?

3. (15 points) Consider a three-card poker game. There are only Ace (A), King (K), and Queen (Q) in a deck where Ace is the highest and Queen is the weakest card. Each player gets exactly one card and each possible allocation is equally likely.

   The game proceeds as follows. Player 1 moves first and chooses whether to fold or bid. If she folds, the game ends with a payoff of -1 for player 1 and 1 for player 2. If she bids, then it is player 2's turn to choose whether to fold or bid. If player 2 folds, then the game ends with a payoff of 1 for player 1 and -1 for player 2. If she bids, then the player with the higher card wins for a payoff of 2, while the losing player gets -2.

   (a) What is the appropriate equilibrium concept to apply to this game?

   (b) Let $p_i(C)$ for $C \in \{A, K, Q\}$ denote the probability of bidding for player $i = \{1, 2\}$ given that the player holds a card C. Does player 1 have a dominant strategy in this game?

   (c) Consider player 2's optimal response to player 1's choice to bid. For each card realization C for player 2, derive player 2's posterior belief regarding player 1's card and the corresponding optimal response $p_2(C)$ given this posterior belief.

   (d) Given the optimal strategy derived in part c), consider player 1's optimal decision for any card realization C. Would player 1 ever choose to bid if she is holding a King? How about a Queen?

4. (20 points) A fishery is supplying fish to a local restaurant. The local restaurant's revenue is

$$\pi(q, \theta) = \theta^2 + 2\theta q - q^2,$$

   where q is the amount of fish supplied by the fishery, and θ is the type of the restaurant. This type θ is known only to the restaurant, but it is common knowledge that $\theta \in \{1, 2\}$ with $\mathrm{Prob}(\theta = 1) = 0.6$, and $\mathrm{Prob}(\theta = 2) = 0.4$. Suppose the fishery has all the bargaining power and makes a menu of contract offers $(q_1, t_1; q_2, t_2)$, where t_i is the total payment made from the restaurant to the fishery if the corresponding contract is accepted. Suppose the outside option of the restaurant is 0. Also assume that there is no cost to produce fish. Thus, the fishery wants to maximize the expectation of the total payment t.

   (a) As a benchmark, suppose that the fishery can observe θ before offering the contracts. What are the optimal contracts $(q_1^*, t_1^*; q_2^*, t_2^*)$?

   From this point on, suppose that the fishery does not observe θ.

   (b) Write down the fishery's optimization problem with all the constraints given that the fishery serves both types of restaurants.

   (c) Derive the optimal contracts for the fishery $(q_1^{**}, t_1^{**}; q_2^{**}, t_2^{**})$ assuming the *usual* constraints bind.

   (d) Derive the information rent earned by the high-type restaurant.

   Now suppose that the owner of the fishery does not want to supply his fish to a low-type restaurant ($\theta = 1$). So instead of making a menu of two contract offers, he simply makes one offer (q, t), aiming only for the high-type restaurant ($\theta = 2$).

   (e) Derive the optimal contract $(\widehat{q}, \widehat{t})$.

5. (15 points) Consider the two-commodity two-agent exchange economy where the set of agents is $N = \{1, 2\}$; consumption spaces are the set of non-negative bundles denoted by (x, y); and their endowments are $\omega_1 = (1, 0)$ and $\omega_2 = (0, 1)$, respectively for agents 1 and 2. Agents' preferences are represented by utility functions $u_1(x, y) = x^a y^{1-a}$ where $a \in (0, 1)$, and $u_2(x, y) = x + y$. At the unique competitive equilibrium outcome of this economy $(x_1^*, y_1^*), (x_2^*, y_2^*))$, the gains from trade assigned to agent 2, i.e.,

$$\frac{u_2(x_2^*, y_2^*) - u_2(\omega_2)}{u_2(\omega_2)} \times 100\%,$$

is equal to?

   (a) Define competitive equilibrium

   (b) Draw the offer curves in an Equilibrium box

   (c) Find the equilibrium and calculate the gains from trade

6. (15 points) Suppose that an agent has wealth $w > 0$ and has the possibility to invest in two different assets: a risk free asset with return $r > 0$ (if the agent invest x in this asset, receives back rx); and a risky asset. The risky asset returns $k > r$ with probability $a \in (0, 1)$, and returns zero with probability $(1 - a)$ (if the agent invests x in the risky asset receives back kx with probability a and 0 with probability $(1 - a)$). Assume that wealth that is not invested in any of the two assets receives return zero. Denote by $0 \leq x \leq w$ the amount that is invested in the risk free asset.

   If the agent is an expected utility maximizer with utility index $u(z) = \sqrt{z}$, then what is the agent's optimal portfolio problem?

   (a) Set the agent's optimal portfolio problem

   (b) Solve the agent's optimal portfolio problem

   (c) Interpret the agent's optimal portfolio problem