PhD Qualifier Examination

Department of Agricultural Economics

June 4, 2021

Instructions

This exam consists of **six** questions. You must answer all questions. If you need an assumption to complete a question, state the assumption clearly and proceed. Be as clear as possible in your answer. You have four hours to complete the exam. Show all your work. If necessary, use math, graphical analysis and provide definitions of key concepts.

- Be sure to put your assigned letter and no other identifying information on each page of your answer sheets.
- Also, put the question number and answer page number (e.g. 4-1) at the top of each page.
- Write on only one side of your paper and leave at least 1 inch margins on all sides.
- Make sure your writing is clear and easy to read.
- Turn in your final copy with all pages in order.

GOOD LUCK!

1. (15 points) Suppose Y is an $n \times 1$ vector and X is an $n \times d$ matrix. The pseudo-inverse of X is defined as

$$X^{+} = (X^T X)^{-1} X^T$$

Consider the linear regression model

$$Y = X\beta + u$$

where β is a $d \times 1$ vector of coefficients to be estimated, and u is an $n \times 1$ vector of iid errors with mean zero and finite variance.

   (a) Derive the OLS estimator $\hat{\beta}$ using the pseudo-inverse of X.

   (b) What condition(s) are required such the estimator derived in part (a) is consistent.

   (c) Suppose $n = d$ and X has a full rank. Show that in this case, the OLS estimator $\hat{\beta} = X^{-1}Y$. What is the R^2 for this estimator?

2. (20 points) A cross-section data set contains the following variables in each column:

$$lwage : \text{monthly earnings in logarithm}$$
$$educ : \text{years of education}$$
$$exper : \text{years of work experience}$$
$$tenure : \text{years with current employer}$$
$$black : \text{dummy variable}, = 1 \text{ if black}, = 0 \text{ otherwise}$$
$$age : \text{age in years}$$
$$sibs : \text{number of siblings}$$
$$meduc : \text{mother's education in years}$$

Consider the following linear regression model to study the return to education:

$$lwage = \beta_0 + \beta_1 educ + \beta_2 exper + \beta_3 tenure + \beta_4 black + \beta_5 age + \epsilon$$

   (a) Explain why $educ$ is endogenous in the above model.

   (b) Suppose $educ$ is endogenous and all other independent variables are exogenous. Can you have an unbiased and consistent estimate of β_1 using OLS? Can you have unbiased and consistent estimates of β_2 and β_3 with OLS? Explain.

   (c) Explain why $sibs$ and $meduc$ can be used as proper instruments for $educ$. Describe how to test if $sibs$ and $meduc$ are legitimate IVs for $educ$ in this model.

   (d) According to the IV estimation results using $sibs$ and $meduc$ as IVs, $\hat{\beta}_1 = 0.083$ and $\hat{\beta}_4 = -0.12$. Both estimates have p-values less than 0.001. What are the interpretations of $\hat{\beta}_1$ and $\hat{\beta}_4$?

   (e) Design a test to examine if $educ$ is indeed endogenous. Briefly explain how to implement the test.

   (f) How do you test for the presence of heteroskedasticity in this model? If heteroskedasticity is identified, how to account for it in your estimation results?

3. (15 points) Consider the extensive-form game shown in Figure 1.

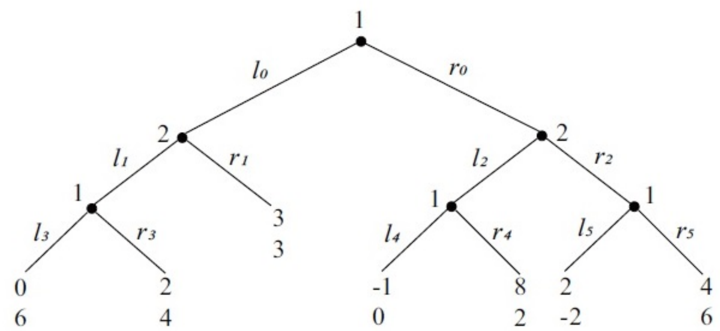

Figure 1

(a) State the rationality/knowledge assumptions necessary for each step in the backward induction process.

(b) Write the game in normal form.

(c) Find all the rationalizable strategies in this game using the normal form of the game. State the rationality/knowledge assumptions necessary for each elimination.

(d) Find all the Nash equilibria in this game.

(e) Find the pure strategy subgame perfect Nash equilibrium in this game.

4. (20 points) Consider a game with two players. Player 1 chooses from three strategies: U, V, and W. Player 2 chooses from two strategies: L and R. Player 2 only knows whether player 1 has chosen U while making her decision and does not know any other information. In the case in which player 1 has chosen U, if player 2 then chooses L, and their payoff profiles will be $(0, 2)$, where 0 is player 1's payoff profile and 2 is player 2's payoff profile. If player 2 then chooses R, their payoff profiles will be $(2, 0)$. Similarly, if these two players choose V and L one after another, their payoff profiles will be $(-1, -1)$; if these two players choose V and R one after another, their payoff profiles will be $(3, 0)$. If these two players choose W and L one after another, their payoff profiles will be $(-1, -1)$; if these two players choose W and R one after another, their payoff profiles will be $(2, 1)$.

(a) Represent this game in extensive-form.

(b) Find all pure strategy Nash equilibria of this game.

(c) Find all pure strategy weak perfect Bayesian equilibria of this game.

(d) Find out all pure strategy sequential equilibria of this game and compare them with the results in part (b).

5. (15 points) Let $X = \{x, y, z\}$. Consider the family of sets $\mathcal{B} = \{\{x, y\}, \{x, z\}, \{z, y\}, \{x, y, z\}\}$. Suppose that you ask an agent to choose an alternative from each set. The agent has a utility function $u(x) = 0$, $u(y) = 10$, $u(z) = 11$. Then in each time the agent chooses, three random variables are realized, e_x, e_y, e_z. These variables are i.i.d uniformly on [-2,2]. The agent chooses the available alternative that maximizes $u(x) + e_x$, $u(y) + e_y$, and $u(z) + e_z$. Suppose

that you ask the agent to choose only once from each set(the random variables are drawn each time that you ask the agent to choose). Let $c = (\mathcal{B}, C)$ be the corresponding choice structure.

(a) Define strong rationalizability by a complete and transitive preference relation of a choice structure.

(b) When is a choice structure strongly rationalizable?

(c) What is the probability that c is strongly rationalizable by a preference relation?

6. (15 points) Consider the utility function on $\mathbb{R}^2$, $u(x, y) = \min\{x^2y, xy\}$. What is the Slutsky matrix of the demand associated with this function for prices $p_x = 1$ and $p_y = 5$ and $w = 10$?