PhD Qualifier Examination

Department of Agricultural Economics

May 31, 2022

Instructions

This exam consists of **six** questions. You must answer all questions. If you need an assumption to complete a question, state the assumption clearly and proceed. Be as clear as possible in your answer. You have four hours to complete the exam. Show all your work. If necessary, use math, graphical analysis and provide definitions of key concepts.

- Be sure to put your assigned letter and no other identifying information on each page of your answer sheets.
- Also, put the question number and answer page number (e.g. Q4–p1) at the top of each page.
- Write on only one side of your paper and leave at least 1 inch margins on all sides.
- Make sure your writing is clear and easy to read.
- Turn in your final copy with all pages in order.

GOOD LUCK!

1. (15 points) Consider the case of a linear regression model. There is a regression model

$$y = X\beta_0 + u, \ u \sim \mathrm{IID}(0, \sigma_0^2 \mathbf{I}), \tag{1}$$

and another regression model

$$y = X\beta + Z\gamma + u, \ u \sim \mathrm{IID}(0, \sigma^2 \mathbf{I}). \tag{2}$$

In the above two models, y is an $n \times 1$ vector, X is an $n \times k_1$ data matrix, β is a $k_1 \times 1$ vector, Z is an $n \times k_2$ data matrix, γ is a $k_2 \times 1$ vector, and u is an independent and identically distributed (IID) random variable of error term. For the following questions, make assumptions if needed.

   (a) Suppose you estimate model 2 while the data are actually generated by model 1. This is the case of over-specification. What will happen to the estimate of β? You need to derive $\hat{\beta}$ and compare with β_0 in model 1. Does $E(\hat{\beta}) = \beta_0$ still hold?

   (b) Following the above question, what will happen to the variance of $\hat{\beta}$, compared to the variance of β_0? Note that you need to derive the variance estimator of $\hat{\beta}$.

   (c) Suppose you estimate model 1 while the data are actually generated by model 2. Note this is the opposite case of the first question, called under-specification. Sometimes it is also called the omitted-variable case. Now the data generation process, or the true model, is model 2. What will happen to the estimate of β_0? You need to derive $\hat{\beta}_0$ and compare with β in model 2. Does $E(\hat{\beta}_0) = \beta$ hold? Explain.

2. (15 points) Suppose you are given an i.i.d. sample $\{Y_i, X_i\}_{i=1}^n$, where X_i is a k-dimensional covariates, and $Y_i > 0$ is the dependent variable. Consider the following model

$$Y_i = \exp(X_i'\beta) + e_i,$$

where e_i is an i.i.d. error with mean zero and finite variance σ^2.

   (a) Propose an estimator for β.

   (b) Derive the asymptotic covariance matrix of your β estimate.

   (c) Construct a test for the hypothesis $H : \beta_1 = 1$.

3. (15 points) (Redacted)

4. (20 points) (Redacted)

5. (15 points) Consider a standard moral hazard problem. A risk-neutral principal hires a risk-averse agent to work for him. Agent's reservation utility is normalized to 0. Let $x = e + \epsilon$ be the output as a function of the agent's effort, e, and an exogenous shock $\epsilon \sim \mathcal{N}(0, \sigma^2)$. The principal offers linear contract $w(x) = a + bx$ and the agent's utility function is of the form

$$U(w, e) = E(w) - r\frac{\sigma_w^2}{2} - \frac{e^2}{2}$$

where r is the coefficient of absolute risk aversion and σ_w^2 is the variance of the wage offer. The agent's outside option is normalized to 0.

   (a) Suppose that the principal can observe and verify the agent's effort. Derive the corresponding first best linear contract.

(b) Suppose that the agent's effort is not observable. Derive the second best linear contract.

(c) How do the first best and second best contracts depend on the agent's risk aversion r and the output variance σ? What is the welfare loss from unobservable effort?

6. (20 points) Consider a seller who wants to sell **two** identical and indivisible items to **three** potential buyers. Each buyer has a unit demand for the item with a valuation for the object v that is independently and identically distributed according to $U \sim [0, 1]$. The distribution of the valuations is common knowledge, but each buyer is privately informed about his valuation. The seller needs to decide whether to conduct a simultaneous or sequential auction. Consider first a simultaneous first-price auction with 0 reserve price and 0 entry fee, in which the highest two bidders win one unit of the item. Suppose that each bidder bids according to a symmetric and strictly increasing bid function $b^*(v)$.

(a) If each of the other bidders bids according to $b^*(v)$, what is bidder i's probability of winning one of the items in the auction given his bidding strategy $\tilde{b}_i(v)$?

(b) What is the expected payoff for bidder i if he bids according to $\tilde{b}_i(v)$?

(c) Solve for the symmetric equilibrium bidding strategy $b^*(v)$.

(d) What is the seller's expected payoff from this simultaneous auction.

Suppose now that instead of a simultaneous auction, the seller decides to conduct two separate first-price auctions to sell each of the items. The auctions take place sequentially. In the first auction, all three bidders participate. The winner of this auction leaves the market and thus the second auction has only two participants. Moreover, suppose that the bidders are **unsophisticated**. That is, they treat each auction as independent. In other words, they are unaware of the possibility of the second auction when they are choosing their bidding strategy in the first auction and they fail to update their beliefs about the opponent's valuation in the second auction.

(e) Consider the second auction and let $b_2^{**}(v)$ denote the symmetric bidding strategy in this first price auction with two bidders. Derive $b_2^{**}(v)$.

(f) Consider the first auction and let $b_1^{**}(v)$ denote the symmetric bidding strategy in this first price auction with all three bidders. Derive $b_1^{**}(v)$.

(g) Calculate the seller's expected payoff from the sequential auction.