PhD Qualifier Examination

Department of Agricultural Economics

July 27, 2022

Instructions

This exam consists of **six** questions. You must answer all questions. If you need an assumption to complete a question, state the assumption clearly and proceed. Be as clear as possible in your answer. You have four hours to complete the exam. Show all your work. If necessary, use math, graphical analysis and provide definitions of key concepts.

- Be sure to put your assigned letter and no other identifying information on each page of your answer sheets.
- Also, put the question number and answer page number (e.g. Question <u>4</u> Page <u>1</u> of <u>3</u>) at the top of each page.
- Write on only one side of your paper and leave at least 1 inch margins on all sides.
- Make sure your writing is clear and easy to read.
- Turn in your final copy with all pages in order.

GOOD LUCK!

1. (15 points) Consider a linear regression model

$$y = X\beta + u,\ E[u \mid X] = 0,\ E[uu' \mid X] = \Omega \neq \sigma^2\mathbf{I}, \tag{1}$$

where y is an $n \times 1$ vector, X is an $n \times k$ data matrix, β is a $k \times 1$ vector, u is a random error term, and Ω is an $n \times n$ variance-covariance matrix with off-diagonal elements equal to 0. For the following questions, state assumptions if needed.

   (a) How does the condition $E[uu' \mid X] = \Omega \neq \sigma^2\mathbf{I}$ impact the expectation of the OLS estimator of β? Is it still unbiased? Show your results.

   (b) Propose a test for heteroskedasticity in the above model. Briefly explain how you are going to realize the test in your empirical work.

   (c) Suppose Ω is completely unknown and the above heteroskedasticity test shows significance, what is the treatment in your empirical work?

   (d) Suppose we believe Ω has a certain form of structure and the above heteroskedasticity test shows significance, what is the treatment?

2. (15 points) Consider the following process, for $t = 1, \ldots, T$,

$$\begin{aligned} e_t &= \rho e_{t-1} + u_t, \\ E[u_t] &= 0, \\ Var[u_t] &= \sigma_u^2, \\ Cov[u_t, u_s] &= 0 \text{ for } t \neq s. \end{aligned}$$

   (a) Derive the mean and variance of e_t. State clearly the condition(s) needed for your derivation.

   (b) Suppose we have a model given by

$$Y_t = X_t\beta + e_t, t = 1, \ldots, T, \tag{2}$$

   where e_t is given above and $E[u_t|X_t] = 0$. Propose an efficient estimator of β. Provide the necessary steps for your estimator.

   (c) Present a test for the hypothesis $H_0 : \rho = 0$ the model in (2). Provide the necessary steps in the construction of your test statistic.

   (d) Again consider the process $e_t = \rho e_{t-1} + u_t$ under the same conditions given above. Suppose it is also known that $\rho = 1$. Derive the mean and variance of e_t.

3. (15 points) (redacted)

4. (15 points) (redacted)

5. (20 points) Consider a three-card poker game. There are only Ace (A), King (K), and Queen (Q) in a deck where Ace is the strongest and Queen is the weakest card. Each player gets exactly one card and each possible allocation is equally likely.

   The game proceeds as follows. Player one moves first and chooses whether to fold or bid. If she folds, the game ends with a payoff of -1 for Player one and 1 for Player two. If she bids, then it is Player two's turn to choose whether to fold or bid. If Player two folds, then the game ends with a payoff of 1 for Player one and -1 for Player two. If she bids, then the player with the stronger card wins for a payoff of 2, while the losing player gets -2.

(a) What is the appropriate equilibrium concept to apply to this game?

(b) Let $p_i(C)$ for $C \in \{A, K, Q\}$ denote the probability of bidding for player $i = \{1, 2\}$ given that the player holds a card C. Does Player one have a dominant strategy in this game for some C?

(c) Consider Player two's optimal response to Player one's choice to bid. For each card realization C for Player two, derive Player two's posterior belief regarding Player one's card and the corresponding optimal response $p_2(C)$ given this posterior belief.

(d) Given the optimal strategy derived in part c), consider Player one's optimal decision for any card realization C. Would Player one ever choose to bid if she is holding a King? How about a Queen?

6. (20 points) Consider a seller who wants to sell **two** identical and indivisible items to **three** potential buyers. Each buyer has a unit demand for the item with a valuation for the object v that is independently and identically distributed according to $U \sim [0, 1]$. The distribution of the valuations is common knowledge, but each buyer is privately informed about his valuation. The seller needs to decide whether to conduct a simultaneous or sequential auction. Consider first a simultaneous first-price auction with 0 reserve price and 0 entry fee, in which the highest two bidders win one unit of the item. Suppose that each bidder bids according to a symmetric and strictly increasing bid function $b^*(v)$.

(a) If each of the other bidders bids according to $b^*(v)$, what is bidder i's probability of winning one of the items in the auction given his bidding strategy $\tilde{b}_i(v)$?

(b) What is the expected payoff for bidder i if he bids according to $\tilde{b}_i(v)$?

(c) Solve for the symmetric equilibrium bidding strategy $b^*(v)$.

(d) What is the seller's expected payoff from this simultaneous auction.

Suppose now that instead of a simultaneous auction, the seller decides to conduct two separate first-price auctions to sell each of the items. The auctions take place sequentially. In the first auction, all three bidders participate. The winner of this auction leaves the market and thus the second auction has only two participants. Moreover, suppose that the bidders are **unsophisticated**. That is, they treat each auction as independent. In other words, they are unaware of the possibility of the second auction when they are choosing their bidding strategy in the first auction and they fail to update their beliefs about the opponent's valuation in the second auction.

(e) Consider the second auction and let $b_2^{**}(v)$ denote the symmetric bidding strategy in this first price auction with two bidders. Derive $b_2^{**}(v)$.

(f) Consider the first auction and let $b_1^{**}(v)$ denote the symmetric bidding strategy in this first price auction with all three bidders. Derive $b_1^{**}(v)$.

(g) Calculate the seller's expected payoff from the sequential auction.