PhD Qualifier Examination

Department of Agricultural Economics

July 31, 2017

Instructions

This exam consists of **six** questions. You must answer all questions. If you need an assumption to complete a question, state the assumption clearly and proceed. Be as clear as possible in your answer. You have four hours to complete the exam. Show all your work. If necessary, use math, graphical analysis and provide definitions of key concepts.

- Be sure to put your assigned letter and no other identifying information on each page of your answer sheets.
- Also, put the question number and answer page number (e.g. 4-1) at the top of each page.
- Write on only one side of your paper and leave at least 1 inch margins on all sides.
- Make sure your writing is clear and easy to read.
- Turn in your final copy with all pages in order.

GOOD LUCK!

1. (15 points) Consider the following model

$$Y_t = \rho Y_{t-1} + u_t, t = 1, 2, \ldots, T$$

where $E[u_t] = 0$.

   (a) Suppose that $\text{var}(u_t) = \sigma^2 < \infty$ and $E[u_t u_s] = 0$ for $t \neq s$. Show that the variance of Y_t is finite only if $-1 < \rho < 1$.

   (b) Consider now a more general model

   $$Y_t = \rho Y_{t-1} + X_t \beta + u_t, t = 1, 2, \ldots, T$$

   Suppose that $u_t = \gamma u_{t-1} + \epsilon_t$, where $E[\epsilon_t] = 0, \text{var}(\epsilon_t) = v^2 < \infty$ and $E[\epsilon_t \epsilon_s] = 0$ for $t \neq s$. Also assume that $|\rho| < 1$ and $|\gamma| < 1$. Suggest a consistent estimator for this model.

2. (20 points) Consider a panel data model

$$y_{it} = x_{it}\beta + c_i + u_{it}, i = 1, \ldots, N, t = 1, \ldots, T.$$

   (a) Under what conditions can this model be consistently estimated with the random effects estimator?

   (b) Derive the variance covariance matrix of the random effects estimator.

   (c) Describe how to estimate the variance covariance matrix proposed in part (b).

   (d) The covariance matrix of v_i is defined as $\Omega = E[v_i v_i']$. Instead of assuming the covariance structure in the standard random effects model, one can estimate Ω with $\hat{\Omega} = 1/N \sum_{i=1}^{N} \hat{\hat{v}}_i \hat{\hat{v}}_i'$, where $\hat{\hat{v}}_i$ is a consistent estimate of v_i. Discuss the statistical properties of the generalized least squares estimator with this alternative covariance matrix. Discuss the pros and cons of this estimator relative to the standard random effects estimator.

3. (15 points) Two political candidates $(i = 1, 2)$ are choosing their campaign positions (p_1 and p_2 respectively). Their positions are modeled as their chosen location on the real line in the interval $[0, 1]$. Their objective is to attract as many votes as possible. Thus, the payoff of candidate i, denoted by v_i, is simply the number of votes that candidate i attracts. There is a continuum of voters, whose favorite positions are uniformly distributed on the interval $[0, 1]$. Suppose that the cost of voting is negligible so that all voters choose to cast a vote. They will vote for the candidate who is closer to their preferred position. Suppose that if both candidates choose the same position, then they share the votes equally.

   (a) Write down the payoff for candidate i, $v_i(p_i, p_j)$.

   (b) Consider a pure strategy equilibrium (p_i^*, p_j^*). Can $p_i^* < p_j^*$ be supported as a pure strategy Nash equilibrium? Either characterize such equilibrium or prove that it does not exist.

   (c) Consider a pure strategy equilibrium (p_i^*, p_j^*). Can $p_i^* = p_j^*$ be supported as a pure strategy Nash equilibrium? Either characterize such equilibrium or prove that it does not exist.

   (d) Suppose now that the number of candidates has increased to 3. Each candidate is choosing their location. Write down the payoff function for candidate i, $v_i(p_i, p_j, p_k)$.

   (e) Find a pure strategy Nash equilibrium of the three candidates location game or show that such equilibrium does not exist.

4. (15 points) Consider the moral hazard problem with three possible effort levels $E = \{e_1, e_2, e_3\}$ and two possible revenue levels $\pi_h = 10$ and $\pi_l = 0$. Let the probability of π_h given e be:

$$p(\pi|e) = \begin{cases} 2/3 & \text{if } e = e_1 \\ 1/2 & \text{if } e = e_2 \\ 1/3 & \text{if } e = e_3 \end{cases}$$

The agent's cost of effort is

$$c(e) = \begin{cases} 5/3 & \text{if } e = e_1 \\ 8/5 & \text{if } e = e_2 \\ 4/3 & \text{if } e = e_3 \end{cases}$$

The agent's utility is $u_a(w, e) = \sqrt{w} - c(e)$, while the principal is risk neutral and seeks to maximize the expected profit $\pi - w$ (revenue minus wage).

(a) Find the optimal revenue maximizing contract when effort is observable.

(b) Suppose that effort is unobservable. Show that under unobservable effort, it is impossible to induce effort level e_2.

(c) Find the optimal contract when effort is unobservable.

5. (20 points) Consider a town populated by two agents $\{1, 2\}$. Each of these agents consume non-negative amounts of an infinitely divisible good that we call money (thus the set of prizes here is $Z \equiv \mathbb{R}_+$). Initial wealth levels are w_1 and w_2, respectively. Agent 2 is initially wealthier than agent 1, i.e., $w_2 > w_1$, and $w_1 > 4$. Denote by x an agent's generic final consumption of money. Both agents have preferences on their distributions of final consumptions of money which are represented by the utility function: for each $F \in \Delta(\mathbb{R}_+)$,

$$U(F) \equiv \int_0^{+\infty} x^\sigma dF(x),$$

where $\sigma \in (0, 1)$.

(a) What is a risk averse agent. Formally define it.

(b) Are these agents risk averse?

(c) What is the coefficient of absolute risk aversion? Calculate it for a given value of $x > 0$ for this agent's utility index.

(d) When does an agent exhibit non-increasing risk aversion towards additive risk? Do these agents exhibit this property?

(e) What is the coefficient of relative risk aversion? Calculate it for a given value of $x > 0$ for this agent's utility index.

6. (15 points) There is a criminal organization known as "The Office of Envigado" that operates in the town described in question 5. For simplicity in the notation we will refer to them as "The Office." The members of The Office extort money from the agents of this town using two different strategies (to extort money from somebody means to obtain it by force, threats, or other unfair means.) The first strategy is to threaten an agent with destroying the equivalent of 4 monetary units from her wealth, and requesting a certain amount in order not to do it. If the agent decides to contact law enforcement and watch out for her belongings the probability that the members of The Office are able to commit their crime is $1/2$.

$$\pi_h = b_{1h} + b_{2h}.$$

   (a) What is the maximum amount of money that an agent would be willing to pay The Office in order to avoid the risk of being attacked? (find an expression as a function of the agent's wealth).

   (b) Suppose that $\sigma = 1/2$. Suppose also that The Office has only available this type of extortion scheme and has limited resources. If they can successfully threaten only one agent, who will they target if they want to maximize their revenue? Provide a formal argument.

   (c) Interpret your answer for the preceding question in light of the theory of risk aversion.

   The second strategy The Office has to extract money from the agents is to threaten an agent with kidnapping her for ransom, and requesting a certain amount in order not to do it. If the agent decides to contact law enforcement and watch out for herself the probability that the members of The Office are able to commit their crime is $\beta \in (0, 1)$. If they end up kidnapping the agent, they extract a proportion $\gamma \in (0, 1)$ of her wealth as ransom.

   (d) What is the maximal amount of money that an agent would be willing to pay The Office in order to avoid the risk of being kidnapped? (find an expression as a function of the agent's wealth).

   (e) Suppose that The Office has only available this type of extortion scheme and has limited resources. Assume also that The Office can commit not to kidnap the agent in exchange for the agent's payment. If they can successfully threaten only one agent, who will they target if they want to maximize their revenue generated by the extortion? Provide a formal argument.