PhD Qualifier Examination

Department of Agricultural Economics

May 29, 2018

Instructions

This exam consists of **six** questions. You must answer all questions. If you need an assumption to complete a question, state the assumption clearly and proceed. Be as clear as possible in your answer. You have four hours to complete the exam. Show all your work. If necessary, use math, graphical analysis and provide definitions of key concepts.

- Be sure to put your assigned letter and no other identifying information on each page of your answer sheets.
- Also, put the question number and answer page number (e.g. 4-1) at the top of each page.
- Write on only one side of your paper and leave at least 1 inch margins on all sides.
- Make sure your writing is clear and easy to read.
- Turn in your final copy with all pages in order.

GOOD LUCK!

1. (15 points) Consider the simple linear model

$$Y_i = X_i\beta + u_i, i = 1, \dots, N.$$

where X_i is a scalar with $E[X_i] = 0$ and u_i's are random errors with mean zero and finite variance.

   (a) Derive the least squares estimator of β. Express its expectation in terms of variance and/or covariance that involve X_i and/or Y_i.

   (b) Derive the probabilistic limit of your estimator. State clearly the assumptions needed for your answer.

   (c) Suppose that Var$(u_i|X_i) = \sigma^2$. Derive the asymptotic variance of your estimator.

   (d) Multicollinearity is a common problem in econometrics. One possible remedy to this problem is the so-called ridge regression, which takes the general form

$$\tilde{\beta} = (X'X + \lambda I_k)^{-1}X'Y,$$

   where X is a $(N \times k)$ design matrix, I_k is a k-dimensional identity matrix and λ is a (small) positive number. When $k = 1$ as in our case, this estimator is simplified to

$$\tilde{\beta} = \frac{X'Y}{X'X + \lambda},$$

   Is this ridge estimator consistent? If not, is there any possible benefit of this approach?

2. (20 points) Consider a panel data model

$$Y_{it} = X_{it}\beta + c_i + u_{it}, \quad i = 1, \dots, N, \quad t = 1, \dots, T.$$

where $E[u_{it}|X_{i1}, \dots, X_{iT}, c_i] = 0$ and $E[c_i u_{it}] = 0$ for all i's and t's. Denote by $v_{it} = c_i + u_{it}$ and $v_i = (v_{i1}, \dots, v_{iT})'$. Define

$$\Sigma = E[v_i v_i'].$$

   (a) Propose a feasible generalized least squares estimator (FGLS) that utilizes an estimate of Σ. State the necessary steps to implement your proposed estimator.

   (b) Let Var$(c_i) = \sigma_c^2$. Further assume that

$$\text{Var}(u_{it}|X_{i1}, \dots, X_{iT}, c_i) = \sigma_u^2.$$

   Propose a FGLS estimator under this additional assumption. State the necessary steps to implement your proposed estimator. Does your estimator guarantee that the estimated $\hat{\sigma}_c^2 > 0$? Explain your answer.

   (c) Discuss the relative advantages and limitations of the two estimators proposed above. Give your recommendations on how to select between these two estimators.

   (d) Suppose that $N = 500$ and $T = 10$. The results based on your proposed estimators from Part (a) and (b) are considerably different. Which estimator would you recommend? Why?

3. (15 points) Consider a standard moral hazard problem. A risk-neutral principal hires a risk-averse agent to work for him. Agent's reservation utility is normalized to 0. Let $x = e + \epsilon$ be the output as a function of agent's effort, e, and an exogenous shock $\epsilon \sim \mathcal{N}(0, \sigma^2)$. The principal offers linear contract $w(x) = a + bx$ and the agent's utility function is of the form

$$U(w, e) = E(w) - r\frac{\sigma_w^2}{2} - \frac{e^2}{2}$$

where r is the coefficient of absolute risk aversion and σ_w^2 is the variance of the wage offer. The agent's outside option is normalized to 0.

(a) Suppose that the principal can observe and verify the agent's effort. Derive the corresponding first best linear contract.

(b) Suppose that the agent's effort is not observable. Derive the second best linear contract.

(c) How do the first best and second best contracts depend on the agent's risk aversion r and the output variance σ? What is the welfare loss from unobservable effort?

4. (15 points) A seller wants to auction off a single item to two bidders. The valuation of each bidder is an iid draw from a uniform distribution on $[0, 1]$. The seller has zero valuation for the item.

(a) Suppose that the seller conducts a second price auction with a reserve price r. Derive a symmetric strictly increasing bid strategy $b^s(v)$ in this auction that constitutes a BNE.

(b) Suppose that the seller conducts a first price auction with a reserve price r. Derive a symmetric strictly increasing bid strategy $b^f(v)$ that constitutes a BNE.

(c) Show the revenue equivalence of these two auctions for the seller. Find the optimal reserve price that maximizes the seller's revenue.

5. (15 points) For $\{\alpha, \beta\} \subseteq (0, 1)$, let f_α and f_β be the functions on $\mathbb{R}^2_+$ defined by $f_\alpha(x) \equiv x_1^\alpha x_2^{(1-\alpha)}$ and $f_\beta(x) \equiv x_1^\beta x_2^{(1-\beta)}$. Consider the binary relation B on $\mathbb{R}^2_+$ defined as follows: for each pair $\{x, y\} \subseteq \mathbb{R}^2_+$,

$$x\, B\, y \Leftrightarrow \{f_\alpha(x) \geq f_\alpha(y) \; and \; f_\beta(x) \geq f_\beta(y)\}.$$

(a) For which combinations of α and β is B complete?

(b) For which combinations of α and β is B transitive?

(c) For which combinations of α and β is B continuous?

6. (20 points) Consider a society of n agents $N = \{1, ..., n\}$. There is a finite set of prizes Z available. An agent, say i, has preferences on $\Delta(Z) \times 2^{N\setminus\{i\}}$. That is, an agent's welfare depends on the distribution (lottery) on the set of prizes assigned to her and a subset of other agents who are assigned to consume it with her. The lottery that assigns prize $z \in Z$ with probability one is δ_z. Let p and q be lotteries on prizes and G and H be subsets of $N \setminus \{i\}$. We write

$$(p, G) \succsim_i (q, H)$$

when agent i finds lottery p received with agents G is at least as good as lottery q received with agents H. We write

$$p \succsim_i^G q$$

when $(p, G) \succsim_i (q, G)$. Thus, $\succsim_i^G$ can be seen as a binary relation on $\Delta(Z)$.

(a) Fix $G \subseteq N \setminus \{i\}$. State the axioms on $\succsim_i^G$ that guarantee this binary relation is represented by expected utility, i.e., there is $u_i^G : Z \to \mathbb{R}$ such that for each pair $\{p, q\} \subseteq \Delta(Z)$, $p \succsim_i^G q$ if and only if $\sum_{z\in Z} p(z)u_i^G(z) \geq \sum_{z\in Z} q(z)u_i^G(z)$.

(b) Suppose now that for each $G \subseteq N \setminus \{i\}$, $\succsim_i^G$ is an expected utility preference. Suppose also that $\succsim_i$ satisfies the following axioms.

C0 There are perfectly private-good prizes that are best and worst for the agent. That is, there are $\{z^*, z_*\} \subseteq Z$ such that for any subsets of $N \setminus \{i\}$, G and H, $(\delta_{z^*}, G) \sim_i (\delta_{z^*}, H)$, and $(\delta_{z_*}, G) \sim_i (\delta_{z_*}, H)$; and for each $p \in \Delta(Z)$, $(\delta_{z^*}, G) \succsim_i (p, G) \succsim_i (\delta_{z_*}, G)$.

C1 $\succsim_i$ is complete and transitive.

C2 For any pair (p, G) and (q, H) and each perfectly private lottery, i.e., $r \in \Delta(Z)$ such that for each G and H, $(r, G) \sim_i (r, H)$, we have that $(p, G) \succsim_i (q, H)$ if and only if for each $\alpha \in (0, 1)$, $(\alpha p + (1 - \alpha)r, G) \succsim_i (\alpha q + (1 - \alpha)r, H)$.

Show that there exists an expected utility representation of $\succsim_i$ in the following sense: there is a family of functions $\{u_i^G : G \subseteq N \setminus \{i\}\}$ such that for any (p, G) and (q, H), $(p, G) \succsim_i (q, H)$ if and only if

$$\sum_{z\in Z} p(z)u_i^G(z) \geq \sum_{z\in Z} q(z)u_i^H(z).$$

Proceed as follows:

1. Argue that there is a family of functions $\{u_i^G : G \subseteq N \setminus \{i\}\}$ such that for each $G \subseteq N \setminus \{i\}$, u_i^G is the expected utility index of $\succsim_i^G$, $u_i^G(\delta_{z_*}) = 0$, and $u_i^G(\delta_{z^*}) = 1$.
2. For each (p, G), let $U_i(p, G) = \sum_{z\in Z} p(z)u_i^G(z)$. Show that for each (p, G),
$$(p, G) \sim_i (U_i(p, G)\delta_{z^*} + (1 - U_i(p, G))\delta_{z_*}, G).$$
3. Prove that for each $\lambda \in (0, 1)$,
$$(\lambda\delta_{z^*} + (1 - \lambda)\delta_{z_*}, G) \sim_i (\lambda\delta_{z^*} + (1 - \lambda)\delta_{z_*}, H).$$
4. Prove that for any (p, G) and (q, H), $(p, G) \succsim_i (q, H)$ if and only if
$$\sum_{z\in Z} p(z)u_i^G(z) \geq \sum_{z\in Z} q(z)u_i^H(z).$$