PhD Qualifier Examination

Department of Agricultural Economics

August 3, 2015

Instructions

This exam consists of **six** questions. You must answer all questions. If you need an assumption to complete a question, state the assumption clearly and proceed. Be as clear as possible in your answer. You have four hours to complete the exam. If an answer requires complicated mathematical calculations, students will be given full credit if they simply write down the function that could have been typed into a calculator.

- Be sure to put your assigned letter and no other identifying information on each page of your answer sheets.
- Also, put the question number and answer page number (e.g. 4.1) at the top of each page.
- Write on only one side of your paper and leave at least 1 inch margins on all sides.
- Make sure your writing is clear and easy to read.
- Turn in your final copy with all pages in order.

GOOD LUCK!

1. (15 points) Consider the following panel data model:

$$\text{Income}_{i,t} = \beta_0 + \beta_1 \text{age}_{i,t} + \beta_2 \text{education}_i + \beta_3 \text{gender}_i + c_i + u_{i,t}, \tag{1}$$

for $i = 1, \ldots, N$ and $t = 1, \ldots, T$, where $\text{gender}_i = 1$ if the ith individual is male, and zero otherwise, and c_i is a time invariant individual effect.

   (a) Suppose c_i is correlated with some covariates. What are the possible sources of the correlation? Shall we use the fixed effect or random effect estimator in this case?

   (b) If the fixed effect estimator is used, indicate which coefficient(s) among $\beta_0, \beta_1, \beta_2, \beta_3$ are identified.

   (c) Suppose the fixed effect estimator is employed. How does one estimate possibly time-varying effect of education. Explain how to test if the effect of education is indeed time varying.

   (d) Regressions on variables like income or wages are known to be heteroskedastic with respect to education. At the same time, the linear regression model (1) may produce negative predicted incomes, which are not acceptable. How would you modify the model to avoid these issues?

2. (15 points) Consider the following model:

$$Y_i = \beta_0 + \beta_1 \log(\beta_2 + X_i) + u_i, i = 1, \ldots, N,$$

where $E[u_i|X_i] = 0$, and $X_i > 0$.

   (a) Present an estimator to this model. Describe the necessary steps to solve for the coefficients.

   (b) Derive the variance covariance matrix of the estimated coefficients.

   (c) Present a test for the hypothesis: $\beta_2 = 0$. Describe the necessary steps to carry out the test.

   (d) Suppose X_i and Y_i designate the i-th person's income and consumption expenditure respectively. How do you interpret β_1? What are the possible explanations of β_2?

3. (15 points) Consider a risk-neutral individual (the seller) who owns an item that she values at zero and a risk averse individual (the buyer) who is interested in purchasing the item. Initially, neither party knows the buyer's valuation for the item, v, but they have a common prior that it is exponentially distributed with mean 1, i.e.,

$$Pr(v \leq \tilde{v}) = 1 - e^{-\tilde{v}} = F(\tilde{v}) \; \forall \tilde{v} \in [0, \infty)$$

where $e \approx 2.718$ denotes the base of the natural log. If the buyer pays p for the item and his realized valuation is v, then his *ex post* utility is

$$u(v - p) = 1 - e^{-r(v-p)}$$

where $r > 0$ is a risk aversion parameter (i.e., he has CARA utility). If he does not purchase the item, then his *ex post* utility is $u(0) = 0$.

Suppose the seller has all the bargaining power but that she must use deterministic sales mechanisms (i.e., take-it-or-leave-it offers). The question is whether she should allow the buyer to inspect the item prior to making him an offer. Suppose that if the buyer inspects the item, he privately learns v, and if he does not inspect it, then the parties remain symmetrically uninformed.

(a) If the seller lets the buyer inspect the item and then offers to sell it to him at a price of p, what is the probability that the buyer accepts?

(b) If the seller lets the buyer inspect the item, what offer p^1 will she make? How much expected revenue will the seller earn in this case?

(c) Calculate the buyer's expected utility,

$$\int_0^\infty u(v-p)F'(v)dv,$$

if he is not allowed to inspect the item and he accepts an offer from the seller to buy it for p.

(d) If the seller does not allow the buyer to inspect the item, what offer p^0, will she make? How much expected revenue will the seller earn in this case?

4. (20 points) Consider the moral hazard problem with unobservable effort, a risk neutral principal and a risk averse agent. The agent's utility from exerting effort level $e \in \{e_L, e_H\}$, where $0 < e_L < e_H$, and receiving wage payment $w \geq 0$ is given by:

$$v(w, e) = u(w) - c(e),$$

where $u(\cdot)$ is strictly increasing and strictly concave. The cost of effort $c(e)$ is strictly increasing in e, with $c(0) = 0$. The agent's reservation utility is $\overline{u}$. Output can take three values: $y \in \{y_H, y_M, y_L\}$. The distribution over output signals, y, given effort, is

	y_H	y_M	y_L
e_H	1/2	1/4	1/4
e_L	1/3	1/4	5/12

(a) Suppose that the principal can verify only whether $y = y_H$ or not (i.e., y_M and y_L cannot be distinguished in the contract). Call w_H the wage paid when observing y_H and w_N-the wage paid otherwise. Suppose that the principal wants to induce effort level e_H by the agent. Set up the principal's optimization problem that solves for the optimal contract offer (w_H, w_N) in this case.

(b) Derive the conditions for the optimal contract (w_H, w_L) that induces e_H.

(c) Solve explicitly for the optimal contract (w_H, w_L) when $c(e_H) = 2, c(e_L) = 0, \overline{u} = 43$ and $u(w) = \sqrt{w}$. The optimal contract needs to satisfy a non-negative wage- $w \geq 0$.

(d) Assume now that prior to contracting, the principal may purchase for a price of $P > 0$ an information system that allows the parties to verifiably observe all realisation of the output level y in the table above. Set up the principal's optimization problem that solves for the optimal contract offer (w_H, w_M, w_L) if the principal wants to induce effort level e_H.

(e) Derive the conditions for the optimal contract (w_H, w_M, w_L) that induces effort level e_H.

5. (20 points) Consider a production set $Y \subseteq \mathbb{R}^L$. Let π^Y be the profit function associated with set Y, i.e., for each $p \in \mathbb{R}^L_+$,

$$\pi^Y(p) \equiv \max_{y \in Y} p \cdot y.$$

   (a) State the definition of free disposal and no free lunch. Draw a production set that satisfies these two properties. Illustrate the profit maximization for some price vector p.

   (b) Is π^Y concave, convex, or neither of them? Provide an explicit argument.

   (c) Suppose now that a firm with production set Y may face an uncertain market. The generic distribution of prices in the market is F, with density f. The firm is an expected profit maximizer, i.e., maximizes the function $\mu_{\pi^Y} \equiv \int \pi^Y(p) f(p) dp$. There are two markets in which the firm may participate.

      - Market 1: price is uncertain with some probability distribution.
      - Market 2: price is certain; it is exactly the expected price in Market 1.

   Prove that the firm always finds participating in Market 1 as good as participating in Market 2.

6. (15 points) Consider an exchange economy $((\succsim^i)_{i \in N}, (\omega_i)_{i \in N})$ where preferences are homothetic, continuous, and locally non-satiated (here preferences are on commodity space $\mathbb{R}^L_+$). Let $\Omega = \sum_{i \in N} \omega_i$. Let $p >> 0$ be a given price vector. Suppose that at prices p, each agent's demand is proportional to the aggregate endowment and the summation of demands is equal to the aggregate endowment, i.e., Ω.

   (a) Let $(w_i)_{i \in N} \in \mathbb{R}^N_+$ be an income redistribution vector at prices p, i.e.,

$$\sum_{i \in N} w_i = \sum_{i \in N} p \cdot \omega_i.$$

   Is it feasible to implement $(w_i)_{i \in N} \in \mathbb{R}^N_+$ in a market with transfers, i.e., is there a price vector p' such that if each agent i is given income w_i the summation of demands at price p' is the aggregate endowment?

   (b) Is there an efficient allocation at which no agent is worse off than having her endowment?

   (c) Let $(u_i)_{i \in N}$ be continuous representations of $(\succsim^i)_{i \in N}$. Let $i \in N$ be a given agent and $\bar{u}$ be a value of u_i-utility that is feasible for agent i, i.e., $\bar{u} \in [u_i(0), u_i(\Omega)]$. Show that there is a feasible efficient allocation at which agent i's u_i-utility is $\bar{u}$ and all agents consumption is proportional to the aggregate endowment.