PhD Qualifier
Examination

Department of Agricultural Economics

July 29, 2016

Instructions

This exam consists of **six** questions. You must answer all questions. If you need an assumption to complete a question, state the assumption clearly and proceed. Be as clear as possible in your answer. You have four hours to complete the exam. Show all your work. If necessary, use math, graphical analysis and provide definitions of key concepts.

- Be sure to put your assigned letter and no other identifying information on each page of your answer sheets.
- Also, put the question number and answer page number (e.g. 4-1) at the top of each page.
- Write on only one side of your paper and leave at least 1 inch margins on all sides.
- Make sure your writing is clear and easy to read.
- Turn in your final copy with all pages in order.

GOOD LUCK!

1. (15 points) Consider the following model:

$$Y_i = \beta_0 + \beta_1 X_{1,i} + \beta_2 X_{2,i} + u_i, \quad i = 1, \ldots, N,$$

where $E[u_i|X_{1,i}, X_{2,i}] = 0$ and $\text{var}[u_i|X_{1,i}, X_{2,i}] = \sigma^2 < \infty$. It is also known that $X_{1,i} \perp X_{2,i}$, $E[X_{j,i}] = \mu_j$ and $\text{var}[X_{j,i}] = \sigma_j^2, j = 1, 2$.

   (a) Denote the OLS estimate by $\hat{\beta} = (\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2)$. Derive the probability limit of $\hat{\beta}$ and its asymptotic covariance matrix.

   (b) Let $\hat{\beta}^{(-2)} = (\hat{\beta}_0^{(-2)}, \hat{\beta}_1^{(-2)}, 0)$ be a restricted estimate of the model above with the restriction $\beta_2 = 0$. Derive the probability limits of $\hat{\beta}^{(-2)}$. Discuss the relationship between the probability limit of $\hat{\beta}_1^{(-2)}$ and that of the unrestricted estimate $\hat{\beta}_1$.

   (c) Denote $\hat{\theta} = \hat{\beta}_1^2 + \hat{\beta}_2^2$. Derive the asymptotic variance of $\hat{\theta}$.

2. (20 points) Consider the model:

$$y = \beta_0 + \beta_1 x + \gamma q + v.$$

Suppose we observe an iid sample $\{Y_i, X_i\}, i = 1, \ldots, N$, but no observations of q are available.

   (a) Without observing q, we estimate instead the following model $y = \alpha_0 + \alpha_1 x + u$. Does the OLS estimator of α_1 consistently estimate β_1? Provide a detailed discussion of your answer, especially how your answer may dependent on certain condition(s).

   (b) Suppose we observe a third variable z and $\text{cov}(q, z) \neq 0$. Explain how can we use z in our regression. What is the benefit of using this additional variable? Under this model, is β_1 consistently estimated? Specify the assumptions needed for your answer.

   (c) Suppose instead we observe a third variable w and $\text{cov}(q, w) = 0$. Under what conditions this variable z can be used as an instrument in the regression. Write out your model. Under this model, is β_1 consistently estimated? Specify the assumptions needed for your answer.

   (d) Provide a practical example where the situation described in (b) and (c) occurs. Be specific in the labeling of y, x, q, z, and w.

   (e) Interest lies in the calculation of the elasticity between y and x. Suppose that $\beta_1 = 0.5$, $y = 20$, and $x = 10$. Calculate the elasticity of y with respect to x. What information would this metric convey to the analysts?

3. (20 points) Consider the following game of public good provision. Each of three players must decide whether or not to contribute a fixed amount to a public good, i.e. player $i \in \{1, 2, 3\}$ must choose strategy $s_i \in \{0, 1\}$ where 1 denotes the decision to contribute. The public good is provided if two or more players contribute. Each player earns a benefit v if the good is provided and 0 if it is not. She also incurs a private cost c if she provides the good. This cost is incurred if she contributes, regardless of the contribution decisions of the other players. Assume that $v > c > 0$.

   (a) Solve for all the pure strategy Nash equilibria of this game.

(b) Solve for an equilibrium where one player does not contribute, and the other two players randomize, contributing with a probability z, where $0 < z < 1$. Under what conditions on the parameters does such an equilibrium exist?

(c) Now consider a possible equilibrium where one player contributes for sure, and the other two players randomize. Under what conditions on the parameters does such an equilibrium exist?

(d) Solve for a symmetric mixed strategy Nash equilibrium, where all players contribute with positive probability $\tilde{z}$. Under what conditions on the parameters does such an equilibrium exist?

Consider now a Bayesian game where each player's cost of provision is independently drawn and distributed on $[0, v]$, with a density function $f(\cdot)$, and cdf $F(\cdot)$. Each player is informed about his own cost realization but not that of his opponents.

(e) Consider a symmetric Bayes' Nash equilibrium, in which players follow a cut-off strategy-contribute if $c \leq c^*$ and do not contribute otherwise. What is the expected payoff from contributing for player i, given that the other two players are following the cut-off strategy?

(f) What is the expected payoff from not contributing for player i given that the other two players are following the cut-off strategy?

(g) Solve for the equilibrium value of c^* (implicit solution is fine.)

(h) Suppose that c is drawn from a uniform distribution on $[0, v]$ and solve for c^*.

4. (15 points) Consider a pure exchange economy with $n = 2, X_i = R^2_+, w_1 = (4, 0), w_2 = (2, 4)$. Suppose the consumers have the following utility functions:

$$u_1(x_1, y_1) = x_1 + \frac{y_1}{2};$$

$$u_2(x_2, y_2) = x_2 + y_2$$

(a) Draw the Edgeworth Box with indifference curves and the initial endowment point clearly marked.

(b) Find the set of Pareto efficient allocations in the Edgeworth Box.

(c) Find the demand functions of the two agents.

(d) Draw the offer curves in the Edgeworth Box.

(e) Find all the Walrasian equilibria. Is it unique?

(f) Find the set of core allocations in the Edgeworth Box.

(g) Find the set of strictly fair allocations in the Edgeworth Box.

5. (15 points) Suppose that a firm produces a good with two inputs: labor L and capital K. The firm's production function is:

$$f(L, K) \equiv L^\alpha K^\beta.$$

where $\alpha > 0$ and $\beta > 0$. The price per unit of the good they produce is p and the price per unit of labor is w and of capital r. Denote by x the amount of the good produced by the firm.

(a) What is the cost function associated with a firm? Provide a formal definition.

(b) Calculate the cost function for this particular firm.

(c) What is the profit function of a firm. Provide a formal definition in terms of the cost function in the model in which there is a single output and inputs are L and K.

(d) Obtain restrictions on α and β that guarantee for every $p > 0$, $w > 0$ and $r > 0$, profits are finite for the firm above. What is the value of the firm's profit when parameters guarantee that it is finite for all $p > 0$, $w > 0$ and $r > 0$. Show that when these conditions are satisfied profits are always positive for any $p > 0$, $w > 0$ and $r > 0$. Interpret.

6. (15 points) Suppose now that the firm in question (5) faces an uncertain market in which input prices w and r are deterministic and the output price is random with distribution F as follows: $p = 1$ with probability α and $p = T > 1$ with probability $(1 - \alpha)$ for some $\alpha > 0$. Let $\overline{p}$ be the expected price in this market.

   (a) Will the firm be willing to pay insurance to be guaranteed a market with deterministic price $\overline{p}$ instead of a random price that follows F? Formally justify your answer.

   (b) Will the firm above prefer the distribution F instead of a market with deterministic price $\overline{p}$ for T large enough? Formally justify your answer.