PhD Qualifier Examination

Department of Agricultural Economics

July 27, 2018

Instructions

This exam consists of **six** questions. You must answer all questions. If you need an assumption to complete a question, state the assumption clearly and proceed. Be as clear as possible in your answer. You have four hours to complete the exam. Show all your work. If necessary, use math, graphical analysis and provide definitions of key concepts.

- Be sure to put your assigned letter and no other identifying information on each page of your answer sheets.
- Also, put the question number and answer page number (e.g. 4-1) at the top of each page.
- Write on only one side of your paper and leave at least 1 inch margins on all sides.
- Make sure your writing is clear and easy to read.
- Turn in your final copy with all pages in order.

GOOD LUCK!

1. (20 points) Consider the following linear model:

$$y_i = \beta_0 + \beta_1 x_i + \beta_2 w_i + u_i, i = 1, \ldots, ..., N, \quad (1)$$

where $cov(x_i, u_i) = 0$. Suppose there exists another variable z_i such that $cov(z_i, u_i) = 0$ and $cov(z_i, w_i) \neq 0$.

   (a) Propose a test to examine whether w_i is endogenous in this regression.

   (b) If your test fails to reject the endogeneity of w_i, propose a consistent estimator for model (1).

   (c) Suppose the correlation between w_i and z_i is close to zero (but different from zero), are there any risks of using z_i as an instrumental variable? Explain your answer.

   (d) When exploring the potential strength of an instrument variable as in part (c), one examines the correlation between between w_i and z_i, rather than their covariance. Why is correlation preferred to covariance here?

2. (15 points) Consider the following panel data model

$$\text{Income}_{i,t} = \beta_0 + \beta_1 \times \text{experience}_{i,t} + \beta_2 \times \text{education}_{i,t} + \beta_3 \times \text{gender}_i + c_i + u_{i,t}, \quad (2)$$

for $i = 1, \ldots, N$ and $t = 1, \ldots, T$, where $\text{gender}_i = 1$ if the ith individual is male, and zero otherwise, and c_i is a time invariant individual effect.

   (a) Suppose c_i is correlated with some covariates. What are the possible sources of the correlation? Shall we use the fixed effect or random effect estimator in this case?

   (b) If the fixed effect estimator is used, indicate which coefficient(s) among $\beta_0, \beta_1, \beta_2, \beta_3$ are identified.

   (c) Suppose the fixed effect estimator is employed. How does one estimate possibly time-varying effect of education. Explain how to test if the effect of education is indeed time varying.

   (d) Regressions on variables like income or wages are often heteroskedastic with respect to education. How do you test for the presence of heteroskedasticity in this case? If you do not reject heteroskedasticity, propose an estimator to mitigate heteroskedasticity.

3. (15 points) Consider an agency relationship, in which a principal contracts with an agent. The profit π from the relationship depends on both the agent's effort e and the state of nature $\theta = \{\theta_1, \theta_2, \theta_3\}$ as follows:

states	θ_1	θ_2	θ_3
result when $e = 6$	$\pi = 60,000$	$\pi = 60,000$	$\pi = 30,000$
result when $e = 4$	$\pi = 30,000$	$\pi = 60,000$	$\pi = 30,000$

Both parties believe that the probability of each state is one third. The payoff function of the principal and the agent are as follows:

$$U_p = \pi - w$$

$$U_a = \sqrt{w} - e^2$$

where w is the wage received by the agent. Suppose that the agent will only accept a contract if it gives him an expected utility of at least 114.

(a) Suppose that the agent's effort is contractible. Derive the optimal first best contract from the point of view of the principal $(e^*, w^*(\pi, e^*))$.

(b) Suppose that the agent's effort is not contractible. Derive the optimal wage scheme $w^{**}(\pi, 4)$ from the point of view of the principal that induces effort $e = 4$ by the agent.

(c) Suppose that the agent's effort is not contractible. Derive the optimal wage scheme $w^{**}(\pi, 6)$ from the point of view of the principal that induces effort $e = 6$ by the agent.

(d) Which effort level does the principal prefer when the effort is not observable?

4. (15 points) Consider a linear city model with continuum of consumers uniformly located on the unit interval $[0, 1]$. Each consumer has a unit demand for a good with valuation v (common knowledge). Suppose that firm A is located at 0 and has a marginal cost of production c. For the time being, suppose that firm A is a monopolist. Consumers incur transportation costs $\frac{t}{2}$ per unit distance and need to travel both ways to purchase the good from firm A. Moreover, $v - c \in (\frac{3}{2}t, 2t)$.

(a) Suppose that the monopolist is not able to price discriminate based on location and charges a uniform price p_m to each consumer who purchases the good. Find the price p_m^* that maximizes the monopolist's profit. What is the monopolist's optimal profit $\pi^*(p_m)$? Is the market covered by the monopolist?

(b) Suppose now that the monopolist is able to perfectly price discriminate based on the consumer's location. Let $p_m(x)$ denote the price that the monopolist charges a consumer located at x. What is the optimal price $p_m^{**}(x)$ that maximizes the monopolist's profit at location x.

(c) What is the resulting market profit π_m^{**} under perfect price discrimination? Is the monopolist better off under perfect price discrimination? How about consumers? Provide economic intuition for full credit.

Suppose now that there are two firms competing on the market- firm A located at 0 and firm B located at 1. They engage in Bertrand competition by simultaneously making price offers to consumers.

(d) Suppose that the two firms are not able to price discriminate and instead charge uniform prices p_A and p_B. Derive the Nash equilibrium prices p_A^* and p_B^* of this game. Is the market covered in equilibrium? What are the equilibrium payoffs π_A^* and π_B^*?

(e) What are the Nash equilibrium prices $p_A^{**}(x)$ and $p_B^{**}(x)$ for $x < \frac{1}{2}$. How about $x > \frac{1}{2}$? Is the market covered in equilibrium?

5. (20 points) An academic department has four Ph.D. students $\{1, 2, 3, 4\}$ and four offices to allocate them for the following academic year $\{a, b, c, d\}$. Students are expected utility maximizers. Their preferences over the deterministic assignments of offices, i.e., receiving a given office for sure, are as follows:

R_1	R_2	R_3	R_4
a	a	b	b
c	d	c	d
d	c	d	c
b	b	a	a

The value of each student's utility index for the different offices is as follows: 4 for the preferred office, 3 for the second preferred office, 2 for the third preferred office, 1 for the least preferred office. For instance $u_1(a) = 4$, $u_1(c) = 3$, $u_1(d) = 2$, and $u_1(b) = 1$.

A feasible deterministic allocation is an assignment that gives each student a different office. A feasible random allocation is a probability distribution on the possible deterministic assignments. Each probabilistic assignment determines for each agent a probability distribution on the different offices. A student has expected utility preferences on these probabilistic assignments represented by her expected utility on the distribution of offices that she gets.

The secretary in the academic department will assign the offices as follows. For a given order on the set of students the secretary will call the first student, who will chose her preferred office; then call the second student in the order, who will chose her preferred office among the remaining ones; and so on. Consider the following four orders: $1 \succ^1 3 \succ^1 2 \succ^1 4$; $2 \succ^2 4 \succ^2 1 \succ^2 3$; $1 \succ^3 4 \succ^3 2 \succ^3 3$; and $2 \succ^4 3 \succ^4 1 \succ^4 4$. Here the higher ranked agent is called first to pick her office. For instance, in $\succ^1$, agent 1 picks her office first, agent 3 second, and so on.

(a) What is a Pareto efficient allocation. (Recall that the set of feasible allocations here is the space of random assignments. Thus, define Pareto efficiency in terms of expected utility preferences, so it is easier to answer the following questions. For simplicity you can denote a random assignment as p, a probability distribution on deterministic assignments)

(b) Suppose that the secretary has chosen an order to allocate the offices (a deterministic assignment, perhaps chosen by seniority and grades). To make it concrete, suppose that she will follow $\succ^1$. Is this allocation Pareto efficient (provide a formal argument)?

(c) Now suppose that the secretary announces that she will randomize among the orders that will determine the allocation. Calculate the expected utility, for each agent, of the following two randomizations. (i) orders $\succ^1$ and $\succ^2$ with 1/2 probability each, and (ii) orders $\succ^3$ and $\succ^4$ with 1/2 probability each.

(d) Is randomizing between orders $\succ^1$ and $\succ^2$ with 1/2 probability each Pareto efficient? (Provide a formal argument.)

(e) Does the conclusion in (d) changes if the utility index of the agents is multiplied by a positive constant (provide a formal argument)? Does it change if the utility index is transformed by an arbitrary strictly increasing transformation (provide a formal argument)?

6. (15 points) Consider the binary relation B on $\mathbb{R}^2_+$ defined as follows:

$$(x, y)\, B\, (x', y') \text{ if and only if } ax + by \geq ax' + by' \text{ and } cx + dy \geq cx' + dy',$$

where $a > 0$, $b \geq 0$, $c > 0$, and $d \geq 0$.

(a) For which combinations of a, b, c, d is B complete?

(b) For which combinations of a, b, c, d is B transitive?

(c) For which combinations of a, b, c, d is B continuous?