# PhD Qualifier Examination

## Department of Agricultural Economics

May 26, 2017

### Instructions

This exam consists of **six** questions. You must answer all questions. If you need an assumption to complete a question, state the assumption clearly and proceed. Be as clear as possible in your answer. You have four hours to complete the exam. Show all your work. If necessary, use math, graphical analysis and provide definitions of key concepts.

- Be sure to put your assigned letter and no other identifying information on each page of your answer sheets.
- Also, put the question number and answer page number (e.g. 4-1) at the top of each page.
- Write on only one side of your paper and leave at least 1 inch margins on all sides.
- Make sure your writing is clear and easy to read.
- Turn in your final copy with all pages in order.

GOOD LUCK!

1. (15 points) Consider an iid sample $\{Y_i, X_i\}_{i=1}^n$, where $X_i$ is a $K$-dimensional covariates. Suppose the population model is given by

$$Y_i = X_i\beta + e_i, i = 1, \ldots, n. \tag{1}$$

   (a) Suppose the error term follows a normal distribution with density function

$$f(e) = \frac{1}{\sqrt{2\pi}\sigma}\exp(-e^2/2\sigma^2).$$

   Write out the log likelihood function of the Maximum Likelihood Estimator of model (1). Derive the MLE solution of $\hat{\beta}_{MLE}$. Show that it is equivalent to the Ordinary Least Squares (OLS) estimator $\hat{\beta}_{OLS}$.

   (b) Instead of the normality assumption on the error term given above, suppose that $E[e_i|X_i] = 0$, $E[X_i'X_i]$ is full-ranked, and $\text{Var}[e_i|X_i] = g(X_i)$, where $g : \mathbb{R}^K \to \mathbb{R}^+$ is an unknown non-constant function. How would you estimate $\beta$? Denote your new estimator by $\tilde{\beta}$. Is $\tilde{\beta}$ different from $\hat{\beta}_{MLE}$? How would you estimate the variance covariance of $\tilde{\beta}$?

2. (20 points) We are interested in estimating people's wage as a function of some contributing factors. Suppose we conduct a survey on a random sample of the population, obtaining information on wage and individual characteristics and social economic background. Notice that for those who were not working, their wage information is missing.

   (a) Suppose we drop all observations whose wage information is missing in our wage regression, can we obtain a consistent estimator? Explain your answer.

   (b) Heckman's correction for selection bias is known to provide a consistent estimator for the problem described above. Provide details on (i) the necessary assumptions for his estimator; (ii) step by step instruction on its implementation.

   (c) Explain how to use the estimation results from part (b) to test whether there is selection bias.

   (d) Heckman's original estimator obtains identification via 'functional form' assumption. Discuss its potential consequence and possible remedy.

3. (15 points) Consider a seller who wants to auction off two identical and indivisible items to 3 bidders. Each bidder has unit demand for the item with a valuation for the object $v$ that is independently and identically distributed according to $U \sim [0, 1]$. The distribution of the valuations is common knowledge, but each bidder is privately informed about his valuation. The seller conducts a second price auction, in which the highest two bidders win one unit each and pay the highest losing bid (i.e. the third highest bid).

   Note: The following information will be helpful in your analysis. The density of the $k^{th}$ highest of $n$ iid draws with cdf $F(x)$ and pdf $f(x)$ is given by

$$f_{(k;n)}(x) = \frac{n!}{(n-k)!(k-1)!}[F(x))]^{n-k}[1 - F(x)]^{k-1}f(x)$$

   (a) Consider a symmetric equilibrium, in which the equilibrium bid is strictly increasing in the bidder's valuation. Characterize such bid function $b^S(v)$ in this second price auction.

(b) Given the symmetric bidding strategy $b^S(v)$ you derived in part a), what is the seller's expected payoff from the second price auction?

The seller is considering using a first price auction instead, in which the highest two bidders win one unit each and each winning bidder pays her own bid. In the following parts, you will derive the symmetric and strictly increasing bid function $b^F(v)$ and the corresponding payoff for the seller.

(c) Given that the other bidders bid according to $b^F(v)$, derive an expression for bidder $i$'s expected payoff from bidding $\tilde{b}_i$?

(d) Derive the symmetric equilibrium bidding strategy $b^F(v)$.

(e) Does the revenue equivalence hold in this case? Explain the intuition behind your answer.

4. (15 points) Consider a model of public good provision, in which two donors, $i = \{a, b\}$ are choosing sequentially how much to contribute to a public project. Donor $a$ contributes first and his donation is publicly observable. The quality of the public project is denoted by $v$ and the utility of donor $i$ with wealth $w_i$ from contributing $g_i \in [0, w_i]$ to the public project is given by

$$u_i(w_i, g_i, v) = w_i - g_i + vG^{0.5}$$

where $G = g_a + g_b$ denotes the total money raised for the project. The quality can take one of two values: $v_L = 1$ with probability $\frac{1}{2}$ and $v_H = 3$ with probability $\frac{1}{2}$. Moreover, $w_i > v_H$.

(a) Suppose that the quality of the project is common knowledge. For each $v$, derive donor b's optimal contribution upon observing $g_a$ by the first donor.

(b) Derive the subgame-perfect Nash equilibrium donations $(g_a^*(v), g_b^*(v))$ for each $v$ when the quality of the project is common knowledge.

In what follows suppose that the first donor is privately informed about the quality of the project, but the second donor only knows that the two qualities are equally likely. Thus, we can think of the first donor being one of two types: $v_L = 1$ corresponds to a donor who observes a low realization of the quality and $v_H = 3$ corresponds to a donor who observes a high realization of the quality.

(c) Derive the pooling perfect Bayesian equilibrium that generates the highest donation by donor $a$. Carefully specify a belief structure that supports this pooling equilibrium.

(d) Find the separating equilibrium that maximizes donor $a$'s contribution and satisfies the Cho-Kreps intuitive criterion. Carefully specify a belief structure that satisfies the Cho-Kreps intuitive criterion and supports this separating equilibrium.

5. (20 points) Let $e \equiv ((\succsim^i)_{i\in N}, (Y^j)_{j\in J}, (\omega^i)_{i\in N}, (\theta^i_j)_{i\in N, j\in J})$ be a private ownership economy. Assume that the consumption space of each agent is $\mathbb{R}^L_+$.

(a) Define what a feasible allocation for $e$ is.

(b) Define competitive equilibrium for $e$.

(c) Define what a Pareto efficient allocation for $e$ is.

(d) What is an exchange economy?

(e) State the First Welfare Theorem for $e$.

(f) Prove the First Welfare Theorem. For simplicity restrict it to an exchange economy.

6. (15 points) Two friends $N \equiv \{1, 2\}$ are betting on a race of two horses, "Genuine risk" and "Spectacular Bid." We will simply refer to the horses as G and S and denote the generic horse by $h \in \{G, S\} \equiv H$. Generic agent is $i \in N$. Each agent is a risk neutral expected utility maximizer. There is a race track that manages the agents' bets. For simplicity we will suppose that the race track is not strategic and charges nothing for organizing the race. The race track announces "track probabilities" for the results of the race. This is simply $\pi_G \geq 0$ and $\pi_S \geq 0$ such that $\pi_G + \pi_S = 1$. Given these probabilities the agents bet on the horses and receive a pay that is proportional to the inverse of the horse's probability to win. For instance, if horse $G$'s track probability of winning is $1/2$, the agent receives \$2 for each \$1 bet on $G$ when $G$ wins. The agent receives zero for each \$1 bet on a horse that loses the race. Each agent has a budget $b_i > 0$. Their aggregate budget is one million dollars, i.e., $b_1 + b_2 = 1$ (for simplicity you need to keep this monetary normalization). Let $b_{ih}$ be agent $i$'s bet on horse $h$. Agents have subjective assessments of the probability that each horse wins the race. The probability that agent $i$ assigns to horse $h$ winning the race is $p_{ih}$.

   The race track is interested in having a balanced budget. This can be achieved when the track probabilities *clear the market*, i.e., given $(\pi_G, \pi_S)$, there are bets $(b_{1G}, b_{1S})$ and $(b_{2G}, b_{2S})$ such that each bet maximizes the corresponding agent's expected utility and for each horse $h \in H$,

$$\pi_h = \frac{b_{1h} + b_{2h}}{\sum_{i \in N} b_{iG} + b_{iS}}.$$

   Note that since aggregate budget is $1$, when the agents spend all their budget on bets,.

$$\pi_h = b_{1h} + b_{2h}.$$

   (a) Set up each agent's expected utility maximization problem.

   (b) Suppose that both agents' budget is the same and that agent $1$ believes $G$ will win with probability $1/2$. What probabilities clear the market?

   (c) In what follows we will prove that there are always track probabilities that clear the market. For simplicity assume that each agent believes each horse can win with positive probability. Prove (an argument is accepted) that the following function has a maximum in the set $D \equiv \{x \equiv (x_{ih})_{i \in N, h \in H} : x_{1G} + x_{2G} = 1, x_{1S} + x_{2S} = 1\}$ (here $\log$ means natural logarithm and $\log(0) = -\infty$):

$$\varphi(x) \equiv \sum_{i \in N} b_i \log \left( \sum_{h \in H} p_{ih} x_{ih} \right)$$

   (d) Let $x^* \equiv (x^*_{ih})_{i \in N, h \in H}$ be the solution to the problem in the previous numeral. For each $h \in H$ let

$$\pi^*_h \equiv \max_{i \in N} \left\{ b_i \frac{p_{ih}}{\sum_{h \in H} p_{ih} x^*_{ih}} \right\},$$

   and for each $i \in N$ and each $h \in H$ let

$$b^*_{ih} \equiv x^*_{ih} \pi_h.$$

   Prove that if $x^*_{ih} > 0$ then $\pi^*_h = b_i \frac{p_{ih}}{\sum_{h \in H} p_{ih} x^*_{ih}}$.