PhD Qualifier Examination

Department of Agricultural Economics

May 30, 2023

Instructions

This exam consists of **six** questions. You must answer all questions. If you need an assumption to complete a question, state the assumption clearly and proceed. Be as clear as possible in your answer. You have four hours to complete the exam. Show all your work. If necessary, use math, graphical analysis and provide definitions of key concepts.

- Be sure to put your assigned letter and no other identifying information on each page of your answer sheets.
- Also, put the question number, answer page number, and total number of answer pages for that question (e.g. Question _4_ Page _1_ of _3_) at the top of each page.
- Write on only one side of your paper and leave at least 1 inch margins on all sides.
- Make sure your writing is clear and easy to read.
- Turn in your final copy with all pages **in order.**

GOOD LUCK!

1. (20 points) Consider an agent who has preferences on two-commodity bundles defined as follows: $(x, y)\, B\, (z, w)$ if and only if

$$\min\{x^{\alpha}y^{(1-\alpha)}, x^{\beta}y^{(1-\beta)}\} \geq \min\{z^{\alpha}w^{(1-\alpha)}, z^{\beta}w^{(1-\beta)}\},$$

where $0 < \alpha < \beta < 1$.

   (a) Is B complete and transitive? Is B continuous?

   (b) Characterize an indifference curve of the preference.

   (c) Calculate the demand correspondence associated with this preference. For which price vectors (strictly positive), does it define a demand function?

   (d) Calculate the Slutsky matrix whenever it is well defined.

2. (15 points) Consider money lottery π obtained by tossing a fair coin with payoffs -10 and 10, and lottery π' that is distributed uniformly on the interval $[-v, v]$ for $0 < v < 10$. Determine any stochastic relation between these lotteries.

3. (20 points) Suppose there is a linear regression model given by equation 1:

$$y = X_1\beta_1 + X_2\beta_2 + u, \tag{1}$$

where y is an $n \times 1$ vector, X_1 is an $n \times k_1$ matrix, and X_2 is an $n \times k_2$ matrix. Let X be the $n \times k$ matrix obtained by concatenating X_1 and X_2, i.e., $X = [X_1|X_2]$ and $k = k_1 + k_2$. Let $\hat{\beta}_1$ and $\hat{\beta}_2$ be the OLS parameter estimates from this regression.

Now consider the following regression models, all estimated by OLS:

   (a) $P_1 y = X_2\beta_2 + u$;

   (b) $P_X y = X_1\beta_1 + X_2\beta_2 + u$;

   (c) $M_1 y = X_2\beta_2 + u$;

   (d) $M_1 y = X_1\beta_1 + M_1 X_2\beta_2 + u$;

   (e) $P_X y = M_1 X_2\beta_2 + u$;

Here, P_1 is the projection matrix that projects orthogonally onto the column space of X_1, and M_1 is the matrix given by $M_1 = I - P_1$. Similarly, P_X is the projection matrix that projects orthogonally onto the column space of X.

Using the Frisch–Waugh–Lovell (FWL) theorem and projection matrix notation, check the estimates of β_2 and the residuals for each of the regression models listed above. Then, determine which of the models yield the same estimates of β_2 as the original regression in equation 1, and which of the models have the same residuals as the original regression. Make necessary assumptions as needed.

4. (15 points) Consider the following linear model:

$$y_i = \beta_0 + \beta_1 x_i + \beta_2 w_i + u_i, i = 1, \ldots, ..., N, \tag{5}$$

where $cov(x_i, u_i) = 0$. Suppose there exists another variable z_i such that $cov(z_i, u_i) = 0$ and $cov(z_i, w_i) \neq 0$.

   (a) Propose a test to examine whether w_i is endogenous in this regression.

   (b) If your test fails to reject the endogeneity of w_i, propose a consistent estimator for model (5).

   (c) Suppose the correlation between w_i and z_i is close to zero (but different from zero), are there any risks of using z_i as an instrumental variable? Explain your answer.

   (d) When exploring the potential strength of an instrument variable as in part (c), one examines the correlation between between w_i and z_i, rather than their covariance. Why is correlation preferred to covariance here?

5. (15 points) Consider the following panel data model

$$\text{Income}_{i,t} = \beta_0 + \beta_1 \times \text{experience}_{i,t} + \beta_2 \times \text{education}_{i,t} + \beta_3 \times \text{gender}_i + c_i + u_{i,t}, \tag{6}$$

for $i = 1, \ldots, N$ and $t = 1, \ldots, T$, where $\text{gender}_i = 1$ if the ith individual is male, and zero otherwise, and c_i is a time invariant individual effect.

   (a) Suppose c_i is correlated with some covariates. What are the possible sources of the correlation? Shall we use the fixed effect or random effect estimator in this case?

   (b) If the fixed effect estimator is used, indicate which coefficient(s) among $\beta_0, \beta_1, \beta_2, \beta_3$ are identified.

   (c) Suppose the fixed effect estimator is employed. How does one estimate possibly time-varying effect of education. Explain how to test if the effect of education is indeed time varying.

   (d) Typical regression of income/wage on education suffers endogeneity bias due to self-selection. Could this be an issue for the fixed effects estimator of model (6). Explain your answer.

6. (20 points) Suppose that an individual builder wishes to build a house. The house can be built using combinations of brick (q_1) and wood (q_2) according to the following production function:

$$q_1 q_2 = 1$$

There is one risk-neutral brick supplier with cost $c_1 q_1$ (this is the total cost, not marginal cost!) and one risk-neutral wood supplier with cost $c_2 q_2$, where $c_1 > 0$ and $c_2 > 0$ are parameters. Suppose that the suppliers' reservation profit levels are zero but that the builder must build the house (i.e., her utility if no house is built is $-\infty$). The builder, however, would naturally like to construct the house at the lowest possible cost. Suppose that the builder has all the bargaining power (i.e., she can credibly make a take-it-or-leave-it offers to the suppliers).

(a) Suppose that the builder can observe c_1 and c_2. Derive the optimal input levels (i.e., the conditional factor demand functions), $q_1^*(c_1, c_2)$ and $q_2^*(c_1, c_2)$.

(b) Now suppose that c_i is private information to the supplier i. Suppose also that it is common knowledge that $c_i \in \{c_L, c_H\}$ where $0 < c_L < c_H$. It is also commonly known that $Pr(c_i = c_H) = \lambda$ and that the realizations c_1 and c_2 are statistically independent. Consider a direct symmetric procurement mechanism $\{p(\hat{c}_i, \hat{c}_j), q(\hat{c}_i, \hat{c}_j)\}$ where $p(\hat{c}_i, \hat{c}_j)$ is the monetary payment from the builder to the supplier i and $q(\hat{c}_i, \hat{c}_j)$ is the amount of output i used to build the house (as a function of the reported costs $(\hat{c}_i, \hat{c}_j)$). Let

$$P(\hat{c}_i) = \lambda p(\hat{c}_i, c_H) + (1 - \lambda) p(\hat{c}_i, c_L)$$

be the expected payment to supplier i when he reports $\hat{c}_i$ and supplier j reports truthfully, and let

$$Q(\hat{c}_i) = \lambda q(\hat{c}_i, c_H) + (1 - \lambda) q(\hat{c}_i, c_L)$$

be the corresponding expected quantity of input i. Note that if the suppliers are induced to report truthfully, then the expected cost to the builder can be written as

$$2(\lambda P(c_H) + (1 - \lambda) P(c_L))$$

Use $P(c_H)$ and $Q(c_H)$ to write the individual-rationality constraint for a high-cost supplier.

(c) Use $P(c_L), Q(c_L), P(c_H), Q(c_H)$ to write the incentive-compatibility constraint for a low-cost supplier.

(d) Assume that IR_H and IC_L bind and solve for $P(c_H)$ and $P(c_L)$ from these two constraints.

(e) Assume that the only other binding constraints involve feasibility, i.e. $q(\hat{c}_i, \hat{c}_j) \cdot q(\hat{c}_j, \hat{c}_i) = 1$. Derive the solution $(q^{**}(c_H, c_H), q^{**}(c_L, c_H), q^{**}(c_H, c_L), q^{**}(c_L, c_L))$ (Hint: note that the feasibility and symmetry of the mechanism alone determine $q^{**}(c_H, c_H)$ and $q^{**}(c_L, c_L)$).

(f) Briefly give intuition for why the solution in part e) differs from the one in part a). (What is the nature of the quantity distortion under the second best relative to the first best?)