PhD Qualifier Examination

Department of Agricultural Economics

June 3, 2024

Instructions

This exam consists of **six** questions. You must answer all questions. If you need an assumption to complete a question, state the assumption clearly and proceed. Be as clear as possible in your answer. You have four hours to complete the exam. Show all your work. If necessary, use math, graphical analysis and provide definitions of key concepts.

- Be sure to put your assigned letter and no other identifying information on each page of your answer sheets.
- Also, put the question number, answer page number, and total number of answer pages for that question (e.g. Question _4_ Page _1_ of _3_) at the top of each page.
- Write on only one side of your paper and leave at least 1 inch margins on all sides.
- Make sure your writing is clear and easy to read.
- Turn in your final copy with all pages **in order.**

GOOD LUCK!

1. (20 points) Consider a pure exchange economy with $n = 2$, $X_A = X_B = R^2_+$, $w_A = (0, 10)$, $w_B = (5, 10)$. Suppose the consumers have the following utility functions:

$$u_A(x_A, y_A) = 5x_A y_A - 8$$
$$u_B(x_B, y_B) = 2x_B + 10$$

   (a) Draw the Edgeworth Box with indifference curves and the initial endowment point clearly marked.
   (b) Find the demand functions of the two agents.
   (c) Draw the offer curves in the Edgeworth Box.
   (d) Find the set of Pareto efficient allocations in the Edgeworth box.
   (e) Find all the Walrasian Equilibria. Is it unique?

2. (15 points) Suppose there is a risk-free asset that yields r dollars for every one dollar invested ($r > 1$). Additionally, there is a risky asset where every one dollar invested has a probability of $\frac{1}{2}$ to yield zero dollars and a probability of $\frac{1}{2}$ to yield $3r$ dollars. Let $\precsim_1$ and $\precsim_2$ be the preferences of two investors with utility functions u_1 and u_2, respectively. Both investors are risk-averse, and each has 100 dollars to invest.

   (a) Formulate the utility maximization problem for the investors and determine their optimal investment portfolios.

   (b) Prove that their optimal investment portfolios necessarily involve purchasing a strictly positive amount of the risky asset.

   (c) Prove that if investor 1 is more risk-averse than investor 2, then investor 2 will purchase more of the risky asset than investor 1.

3. (15 points) Suppose that

$$Y_t = X_t + e_t,$$

and

$$X_t = \alpha + \beta X_{t-1} + u_t,$$

where the errors e_t and u_t are mutually independent white noise process. For the following questions, make necessary assumptions if needed.

(a) Show that Y_t is an ARMA(1, 1) process.

(b) Given $X_t = \alpha + \beta X_{t-1} + u_t$, where $u_t \sim WN(0, \sigma^2)$ and assuming $|\beta| < 1$, calculate the mean and variance of X_t.

(c) In empirical studies, how can you determine whether Y_t and X_t are stationary?

4. (15 points) Consider two linear regressions, one restricted and the other unrestricted:

$$y = X\beta + u, \ u \sim \text{IID}(0, \sigma_0^2 \mathbf{I}), \tag{1}$$

and

$$y = X\beta + Z\gamma + u, \ u \sim \text{IID}(0, \sigma^2 \mathbf{I}), \tag{2}$$

where y is an $n \times 1$ vector, X is an $n \times k_1$ matrix, and Z is an $n \times k_2$ matrix.

(a) Show that, in the case of mutually orthogonal regressors, with $X'Z = O$, the estimates of β from the two regressions are identical. Note that O denotes a zero matrix.

(b) Show that the difference between the unrestricted estimator $\tilde{\beta}$ of model 2 and the restricted estimator $\hat{\beta}$ of model 1 is given by

$$\tilde{\beta} - \hat{\beta} = (X'M_Z X)^{-1} X'M_Z M_X y,$$

where M_Z and M_X are projection matrices defined by Z and X respectively.

5. (15 points) Given i.i.d. data $\{y_i, x_i\}, i = 1, \ldots, N$, consider the following two models:

$$y_i = \exp(a_0 + a_1 x_i + u_i), \tag{3}$$

and

$$y_i = \exp(b_0 + b_1 x_i) + v_i, \tag{4}$$

where both u_i and v_i are errors with mean zero and variance one.

(a) Propose an estimator for model (3), provide necessary implementation steps.

(b) Propose an estimator for model (4), provide necessary implementation steps.

(c) Derive the conditional variance $\text{var}(y_i|x_i)$ from these two models and discuss their differences.

6. (20 points) Consider a population with a continuum of risk-neutral citizens of measure one who are choosing whether to get vaccinated against a communicable disease. Contracting the disease is associated with a symmetric cost d. The cost c of getting the vaccine is heterogeneous among the population and is uniformly distributed on $[0, 1]$. Suppose that the probability of contracting the disease is zero for a vaccinated individual and $(1 - \alpha)$ for an unvaccinated individual, where α is the measure of unvaccinated citizens.

   (a) Describe the optimal decision of an individual to get vaccinated as a function of c, d, and α.

   (b) Find the BNE value of α.

   (c) What is the total surplus given the BNE you found in part (b)?

   (d) What is the total expected surplus if all agents with cost $c \leq \hat{c}$ choose to be vaccinated?

   (e) What value of $\hat{c}$ maximizes total surplus?

   (f) Suppose that the government wants to induce the vaccination outcome that maximizes total surplus. Can they accomplish this through a monetary fine imposed on unvaccinated individuals? What should be the fine amount?