PhD Qualifier Examination

Department of Agricultural Economics

July 24, 2024

Instructions

This exam consists of **six** questions. You must answer all questions. If you need an assumption to complete a question, state the assumption clearly and proceed. Be as clear as possible in your answer. You have four hours to complete the exam. Show all your work. If necessary, use math, graphical analysis and provide definitions of key concepts.

- Be sure to put your assigned letter and no other identifying information on each page of your answer sheets.
- Also, put the question number, answer page number, and total number of answer pages for that question (e.g. Question 4 Page 1 of 3) at the top of each page.
- Write on only one side of your paper and leave at least 1 inch margins on all sides.
- Make sure your writing is clear and easy to read.
- Turn in your final copy with all pages **in order.**

GOOD LUCK!

1. (20 points) Suppose that $u(\boldsymbol{x})$ is continuous on $\mathbb{R}_+^L$ and $(\boldsymbol{p}, I) \gg \mathbf{0}$. Prove that the indirect utility function has the following properties:
   (a) $v(\boldsymbol{p}, I)$ is *nonincreasing in* $\boldsymbol{p}$ and *nondecreasing in* I
   (b) $v(\boldsymbol{p}, I)$ *is homogeneous of degree* 0 *in* $(\boldsymbol{p}, I)$
   (c) $v(\boldsymbol{p}, I)$ *is quasi-convex in* $\boldsymbol{p}$. That is, $\{\boldsymbol{p} : v(\boldsymbol{p}, I) \leq k\}$ is a convex set for all k.
   (d) $v(\boldsymbol{p}, I)$ *is continuous.*
   (e) Roy's Identity:

$$x_l(\boldsymbol{p}, I) = -\frac{\frac{\partial v(\boldsymbol{p}, I)}{\partial p_l}}{\frac{\partial v(\boldsymbol{p}, I)}{\partial I}}, \forall l = 1, \ldots, L.$$

2. (15 points) Consider the utility function for an individual: $u(x) = ax^2 + bx + c$.

   (a) What restrictions must be imposed on the coefficients a and b so that the utility function exhibits risk-averse preferences and the individual prefers more wealth for all wealth levels between $[0, \bar{x}]$?

   (b) Prove that u has the property of increasing absolute risk-aversion given the restrictions from part (a).

   (c) Prove that the individual's expected utility function can be determined by the mean and variance of a gamble, given the restricted condition from part (a).

3. (15 points) Let $X_i, i = 1, \ldots, N$, be an I.I.D. sample from a distribution with mean μ and variance $\sigma^2 < \infty$. Consider the following two estimators of μ: $\hat{\mu} = 1/N \sum_{i=1}^{N} X_i$ and $\tilde{\mu} = 1/(N-k) \sum_{i=1}^{N} X_i$, where k is an integer smaller than N.

   (a) Derive the expectation and variance of $\hat{\mu}$

   (b) Derive the expectation and variance of $\tilde{\mu}$

   (c) Derive the mean squared error of these two estimators and show that both are consistent estimators of μ.

4. (15 points) Given data $(X_t, Y_t), t = 1, \ldots, T$, suppose $Y_t = X_t\beta + e_t$, where $e_t = \rho e_{t-1} + u_t$, $E[u_t] = 0$, $\text{Var}(u_t) = \sigma_u^2 < \infty$ and $\text{Cov}(u_t, u_s) = 0$ for $t \neq s$. Define $\boldsymbol{u} = (u_1, \ldots, u_T)^T$ and $\text{Var}(\boldsymbol{u}) = \Omega$.

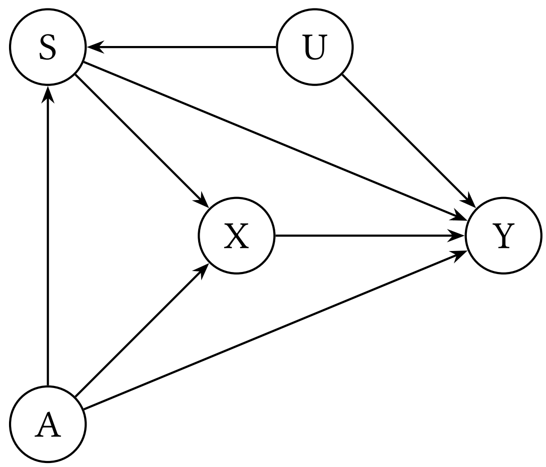
   (a) Derive Ω in terms of σ_u^2 and ρ.

   (b) Show that the model is stationary only if $|\rho| < 1$.

   (c) Propose an asymptotically efficient estimator of β.

   (d) Describe a procedure to test the hypotheses $\beta = \beta_0$.

5. (15 points) Consider a research project aimed at determining the causal effect of HIV (X) on the incidence of stroke (Y). The study incorporates smoking (S) and age (A) as control variables. Additionally, U represents unobservable confounders that influence both smoking and stroke. The hypothetical Directed Acyclic Graph (DAG) for this project is depicted below:



(a) Explain the concept of "backdoor paths" within the context of this DAG. Identify all backdoor paths between X and Y. Discuss whether these paths introduce a confounding bias in the estimation of the causal effect of HIV on stroke.

(b) Applying the "backdoor criterion", suggest an appropriate identification strategy to estimate the total causal effect of X on Y. Present your regression formula. Remember, variable U is unobservable and thus, not available in your dataset. Specify any assumptions necessary for the validity of your proposed strategy.

(c) Suppose the regression formula you propose in the above question includes variable S, can we interpret the coefficient of S as the direct causal effect of smoking on stroke? Explain.

6. (20 points) A monopolistic seller faces a continuum of consumers who have preferences:

$$v(\theta, q, p) = \theta v(q) - p,$$

where q stands for the quantity purchased, θ stands for the willingness to pay (which is heterogeneous in the population), and p stands for the price. Suppose that the quantity can take values $q = \{0, 1, 2\}$ and the corresponding value of consumption is $v(0) = 0$, $v(1) = 1$, and $v(2) = \frac{7}{4}$. The constant per-unit marginal cost is $c = \frac{3}{4}$. There are two types of consumers: $\theta_l = 1$ in proportion λ and $\theta_h = 2$ in proportion $1 - \lambda$. The seller wishes to maximize the expected net profit, the expected revenue minus the expected cost. The outside utility of each buyer is zero.

(a) Suppose that the consumer's type θ is observable. Moreover, suppose that the consumers cannot engage in arbitrage by reselling goods to other consumers or making two individual purchases from the monopolist (perhaps, the monopolist keeps a record and product personalization precludes resale). Derive the optimal sales contract offered by the seller to each consumer type.

For the remaining parts, suppose that the consumer's type θ is private information to the consumer, but the distribution of types in the population is common knowledge.

(b) Suppose that the monopolist could offer a menu of quantity-price pair, i.e., $\{(q = 1, p_1), (q = 2, p_2)\}$ or choose to offer only one of the two available quantities. Moreover, suppose that the consumers cannot engage in arbitrage (as described in part a)). Derive the optimal sales contract by the seller. How does it depend on the proportion of low-valued consumers, λ?

(c) Suppose now that the consumers can engage in arbitrage. That is, if the monopolist offers both quantities, then the consumer who purchases two units will purchase at the lowest of p_2 and $2p_1$. Derive the optimal sales contract by the seller.

(d) How does the arbitrage opportunity change the optimal contract? In particular, how does it affect the two types' consumption and payoffs?