PhD Qualifier Examination

Department of Agricultural Economics

August 2, 2021

Instructions

This exam consists of **six** questions. You must answer all questions. If you need an assumption to complete a question, state the assumption clearly and proceed. Be as clear as possible in your answer. You have four hours to complete the exam. Show all your work. If necessary, use math, graphical analysis and provide definitions of key concepts.

- Be sure to put your assigned letter and no other identifying information on each page of your answer sheets.
- Also, put the question number and answer page number (e.g. 4-1) at the top of each page.
- Write on only one side of your paper and leave at least 1 inch margins on all sides.
- Make sure your writing is clear and easy to read.
- Turn in your final copy with all pages in order.

GOOD LUCK!

1. (15 points) Consider the model $Y_t = X_t\beta + e_t, t = 1, \ldots, T$, where $e_t = \rho e_{t-1} + u_t, E[u_t] = 0$, $\text{Var}(u_t) = \sigma_u^2 < \infty$ and $\text{Cov}(u_t, u_s) = 0$ for $t \neq s$. Let $u = (u_1, \ldots, u_T)^T$ and $\text{Var}(u) = \Omega$.
   (a) Derive Ω in terms of σ_u^2 and ρ.
   (b) Propose a test for the hypothesis $H : \rho = 0$.
   (c) Propose a consistent estimator of Ω.
   (d) Propose an asymptotically efficient estimator of β.

2. (20 points) There is a cross-section data set with a binary response. The variable names are listed below:

$$smoker : \text{1 for smokers and 0 for nonsmokers}$$
$$age : \text{age in years}$$
$$educ : \text{number of years of schooling}$$
$$income : \text{family income}$$
$$pcigs79 : \text{price of cigarettes in individual states in 1979}$$

   (a) Consider a linear probability model for the study:

$$smoker = \beta_0 + \beta_1 age + \beta_2 educ + \beta_3 income + \beta_4 pcigs79 + \epsilon,$$

   where $Pr(smoker = 1|X) = E(smoker = 1|X) = X\beta$. $X\beta$ is the right hand side of the above equation, excluding the error term ϵ. What are the disadvantages of using the linear probability model in empirical work?

   (b) As an alternative, consider using the probit model with latent variable specification:

$$smoker^* = \beta_0 + \beta_1 age + \beta_2 educ + \beta_3 income + \beta_4 pcigs79 + e,\ e \sim N(0,1),$$
$$smoker = 1 \text{ if } smoker^* > 0; \quad smoker = 0 \text{ if } smoker^* \leq 0,$$

   where $N(0, 1)$ is the standard normal distribution. Derive the log-likelihood function of this model. Make assumptions if needed.

   (c) In the probit model, suppose the research interest lies in identifying that if $educ$ and $income$ jointly have significant impact on smoking behavior. Set up the null hypothesis for the research question and explain how to implement the test econometrically.

   (d) Denote the estimate of β_2 from linear probability model as $\hat{\beta}_2$ and that from the probit model as $\tilde{\beta}_2$. According to the estimation results, $\hat{\beta}_2 = -0.021$ and $\tilde{\beta}_2 = -0.056$. Both estimates are statistically significant at $\alpha = 0.01$. How to interpret $\hat{\beta}_2 = -0.021$? How to make $\tilde{\beta}_2$ comparable to $\hat{\beta}_2$ in empirical work?

3. (15 points) Consider a Cournot model in which two firms both produce at constant marginal cost. The market demand is given by $P(Q) = 100 - Q$ with $Q \leqq 100$. Firm 1 has marginal cost equal to $C = 10$ which is common knowledge. Firm 2's marginal cost is private information. It is equal to $C_L = 6$ with probability $\beta = 0.5$ and to $C_H = 10$ with probability $(1 - \beta) = 0.5$.
   (a) Write this formally as a Bayesian game.
   (b) Find a Bayesian-Nash equilibrium.

4. (20 points) Consider the following game:

		buyer B	buyer D
seller	H	$1, 1$	$-1, 2$
	L	$2, -1$	$0, 0$

The seller can choose to work hard (H) or to be lazy (L), and the buyer can choose to purchase (B) or not (D).

(a) Find the set of feasible payoff profiles and the set of individually rational payoff profiles.

(b) Suppose that the game is repeated infinitely, and the discount factor for both players is δ. Find a SPNE such that, for some range of δ, the path of the repeated game is $(H, B)^\infty$. Solve for the range of δ.

(c) Now, suppose that in the stage game, the buyer could observe the effort of the seller prior to making a purchasing decision. Then, solve part (b).

5. (15 points) Consider a researcher who performs the following experiment in a two-commodity economic environment. First, offers the agent to choose an affordable bundle for prices $p = (2, 5)$ and wealth 10. Then, offers the agent to choose an affordable bundle for prices $p' = (5, 2)$ and wealth $w = 10$. Suppose that the agent simply chooses uniformly at random a bundle in the set $M_1 = \{(0, 2), (1, 1.6), (2, 1.2), (3, 0.8), (4, 0.4), (5, 0)\}$ for the first menu. Then, chooses a bundle uniformly at random from $M_2 = \{(2, 0), (1.6, 1), (1.2, 2), (0.8, 3), (0.4, 4), (0, 5)\}$ for the second menu. (The researcher does not observe the process by which the agent chooses; from the point of view the researcher, he or she offers a Walrasian budget set and the agent chooses from it).

(a) What is a choice structure? What is the choice structure associated with this experiment?

(b) Define weak rationalization by means of a preference.

(c) What is a linear preference?

(d) What is the probability that, from the point of view of the researcher, the Choice Structure is weakly rationalized by a linear preference (different from total indifference)?

6. (15 points) Consider a society with n agents who each has a preference on $\mathbb{R}^L_+$. The preference of agent i is represented by utility u_i. An allocation in the economy is a profile of bundles $x = (x^i)_{i=1,...,n}$. At allocation x, agent i receives consumption $x^i \in \mathbb{R}^L_+$. A social planner has a binary relation on allocations in this society. The social planner's binary relation is represented by the following function: $W(x) = W(x^1, ..., x^n) = \min\{u_1(x^1), ..., u_n(x^n)\}$. Provide a formal argument supporting your answer for the following questions.

(a) Is the social planner's binary relation complete?

(b) Is the social planner's binary relation transitive?

(c) Suppose that each u_i is convex. Is the social planner's binary relation convex?