PhD Qualifier Examination

Department of Agricultural Economics

May 31, 2019

Instructions

This exam consists of **six** questions. You must answer all questions. If you need an assumption to complete a question, state the assumption clearly and proceed. Be as clear as possible in your answer. You have four hours to complete the exam. Show all your work. If necessary, use math, graphical analysis and provide definitions of key concepts.

- Be sure to put your assigned letter and no other identifying information on each page of your answer sheets.
- Also, put the question number and answer page number (e.g. 4-1) at the top of each page.
- Write on only one side of your paper and leave at least 1 inch margins on all sides.
- Make sure your writing is clear and easy to read.
- Turn in your final copy with all pages in order.

GOOD LUCK!

1. (15 points) Consider the following panel data model

$$\text{Income}_{i,t} = \beta_0 + \beta_1 \log(\text{age}_{i,t}) + \beta_2 \text{education}_i + \beta_3 \text{gender}_i + c_i + u_{i,t}, \quad (1)$$

for $i = 1, \ldots, N$ and $t = 1, \ldots, T$, where $\text{gender}_i = 1$ if the ith individual is male, and zero otherwise, and c_i is a time invariant individual effect.

   (a) Suppose c_i is correlated with some covariates. What are the possible sources of the correlation? Shall we use the fixed effect or random effect estimator in this case?

   (b) If the fixed effect estimator is used, indicate which coefficient(s) among $\beta_0, \beta_1, \beta_2, \beta_3$ are identified.

   (c) Suppose the fixed effect estimator is employed. How does one estimate possibly time-varying effect of education. Explain how to test if the effect of education is indeed time varying.

   (d) How to modify model (1) to allow time-varying education effects that might differ between males and females?

2. (15 points) Consider the following model

$$Y_i = \beta_0 + \beta_1 \log(\beta_2 + X_i) + u_i, i = 1, \ldots, N,$$

where $E[u_i|X_i] = 0$, and $X_i > 0$.

   (a) Present an estimator to this model. Describe the necessary steps to solve for the coefficients.

   (b) Derive the variance covariance matrix of the estimated coefficients.

   (c) Suppose that β_2 is known to be positive. Propose a way to impose this restriction into estimation.

   (d) Suppose X_i and Y_i designate the i-th person's income and consumption expenditure respectively. How do you interpret β_1? What are the possible explanations of β_2?

3. (15 points) Consider a seller who wants to auction two identical objects. There are three bidders ($i = 1, 2, 3$) participating in the auction and each bidder has an identical and commonly known valuation for the object of v for the first unit and 0 for the second unit. Assume that the seller's valuation is normalized to 0. The seller conducts an all-pay auction, in which the two highest bidders win one unit, but all bidders pay their bids independent of whether they have won the object. Let $F(b)$ denote a symmetric atomless mixed strategy Nash equilibrium CDF of bids in this game.

   (a) From the point of view of bidder i, what is the probability that he loses the auction given that the other two bidders bid according to $F(b)$?

   (b) From the point of view of bidder i, what is the probability that he wins one of the objects given that the other two bidders bid according to $F(b)$?

   (c) Write the expected payoff of bidder i, $\pi_i(b_i)$ given that the other two bidders bid according to $F(b)$?

   (d) Derive the Nash equilibrium CDF $F(b)$ with its corresponding support.

4. (20 points) Suppose a risk-neutral principal faces two identical agents with CARA utility,

$$u_i(y_i) = -\exp(-ry_i), \quad i = 1, 2,$$

where $y_i \in \Re$ is wealth and $r > 0$ is the coefficient of absolute risk aversion. The output of agent i is

$$\pi_i = a_i + \epsilon,$$

where $a_i \geq 0$ is effort and $\epsilon \sim N(0, \sigma^2)$. In other words, the agents face a *common* normally distributed shock to their output. The cost (measurable in monetary units) to agent i of exerting effort a_i is $C(a_i) = 0.5a_i^2$. Each agent's outside option is to take a job that requires no effort and imposes no risk at a wage of $\overline{w} = 0$.

The principal would like to design an optimal linear incentive scheme for each agent i of the form

$$w_i(\pi_i) = m_i\pi_i + b_i,$$

where $m_i \geq 0$ and $b_i \in \Re$.

(a) Derive each agent's expected utility under this linear contract. What is the agent's certainty equivalent wealth? Hint: Recall that if y_i is normally distributed with mean μ and variance v^2, then $E[-exp(-ry_i)] = -exp[-r(\mu - \frac{1}{2}rv^2)]$. You don't need to prove this result.

(b) Write down the principal's optimization program for obtaining the optimal values of m_i^* and b_i^*.

(c) Use the IR and IC constraints to substitute choice variables out of the principal's program and restate it as an unconstrained problem.

(d) Solve for the optimal contract. What is the principal's expected payoff corresponding to this optimal contract? Does this contract expose the agents to any risk?

Suppose now that the principal decides to offer an alternative contract to the two agents of the form

$$w_i(\pi_i, \pi_j) = \overline{m}\pi_i - \underline{m}\pi_j + b_i,$$

where $\overline{m} \geq 0$ and $\underline{m} \leq \overline{m}$ and $b_i \in \Re$. Thus, this new contract makes each agent's wage conditional not only on her own output, but also the output of the other agent.

(e) Given this new incentive scheme, derive agent i's certainty equivalent wealth. Write down agent i's participation (IR) constraint in terms of her certainty equivalent wealth.

(f) What is agent i's incentive constraint (IC)?

5. (20 points) Consider an exchange economy populated by two agents $\{1, 2\}$ whose preferences on consumption bundles of two commodities are represented by the following utility functions: $u_1(x, y) \equiv x^{\frac{1}{2}}y^{\frac{1}{2}}$ and $u_2(x, y) = \min\{x, y\}$. The endowment of agent 1 is $(3, 1)$ and the endowment of agent 2 is $(1, 3)$.

(a) The following is a frame for the Edgeworth box for this economy. Draw agent 1's indifference curve through her endowment. What is the slope of this curve at her endowment?

(b) Normalize the price of commodity one to one. Thus prices in this economy are determined by p_y. Calculate the offer curve of agent 1, i.e., derive the expression of the agent's demand given p_y. Plot this curve in the Edgeworth box. What is the relation of the offer curve and the indifference curve of the agent at her endowment?

(c) Draw agent 2's indifference curve through her endowment.

(d) Draw agent 2's offer curve (include demand for boundary prices).

(e) What is the competitive equilibria of this economy? Calculate and illustrate.

(f) Suppose now that a social planner knows agent 1's preferences and asks agent 2 for her preferences in order to recommend an allocation in this economy. The social planner will calculate the competitive equilibria of the economy and recommend one of the competitive equilibrium outcomes for the reported preferences. Show that agent 2's preferences are actually a report that maximizes her preferences given what the social planner will recommend.

6. (15 points) An investor has wealth $w > 0$ and is considering investing her wealth in two different assets: Tesla stocks and a risk free asset with return $r > 0$. The latest forecast for Tesla stock (for the period in which the risk free asset matures) is that its price will be $\$10$ with 40% chance and will return to its maximum of $\$376.79$ with 60% chance. The current price of Tesla stock is $\$190.53$. The agent is an expected utility maximizer with utility index $u(z) \equiv z^\sigma$ with $\sigma \in (0,1)$.

(a) Define what is a risk averse agent.

(b) Is this agent risk averse?

(c) Write down the optimal portfolio problem for this agent. (denote x the amount of money the agent invests in Tesla stock). Derive an expression characterizing this amount (it is not necessary to solve it).