PhD Qualifier Examination

Department of Agricultural Economics

June 2, 2025

Instructions

This exam consists of **six** questions. You must answer all questions. If you need an assumption to complete a question, state the assumption clearly and proceed. Be as clear as possible in your answer. You have four hours to complete the exam. Show all your work. If necessary, use math, graphical analysis and provide definitions of key concepts.

- Be sure to put your assigned letter and no other identifying information on each page of your answer sheets.
- Also, put the question number, answer page number, and total number of answer pages for that question (e.g. Question 4 Page 1 of 3) at the top of each page.
- Write on only one side of your paper and leave at least 1 inch margins on all sides.
- Make sure your writing is clear and easy to read.
- Turn in your final copy with all pages **in order.**

GOOD LUCK!

1. (15 points) Consider an economy with two consumers and two commodities. Each consumer's consumption set is $\mathbb{R}^2_+$. Consumers' utility functions are:

$$u_1(x_1, y_1) = \min\{2x_1, y_1\}$$
$$u_2(x_2, y_2) = x_2 y_2$$

The total endowment in the economy is $(x, y) = (1, 3)$.

(a) Find **all** Pareto optimal allocations for this economy.

(b) Find all Pareto optimal allocations at which the two consumers have the same levels of utility.

(c) Suppose that the initial endowments are $w_1 = (1, 0)$ and $w_2 = (0, 3)$, derive the demand function.

(d) Solve for the competitive equilibrium.

2. (15 points) An individual has the Bernoulli utility function $u(x) = -\frac{1}{x}$ (for $x > 0$). Suppose that the individual faces a gamble: receiving x_1 with probability p and losing x_2 with probability $1 - p$.

   (a) For what level of initial wealth w_0 is the individual willing to accept this gamble?

   (b) Suppose that $x_1 = x_2 < w_0$. What is the probability level such that the individual will accept the gamble at each initial wealth level? Based on this fact, can you hypothesize whether the individual's absolute risk aversion coefficient is increasing or decreasing?

   (c) Prove your conjecture from question 2.

3. (15 points) Let x be a random variable from the Poisson distribution with coefficient $\lambda > 0$. The probability of x is given by

$$P(x = X) = \frac{\lambda^X e^{-\lambda}}{X!},\ X \in (0, 1, 2, ...)$$

Suppose you are given an i.i.d. sample $X_i, i = 1, .., N$, from this distribution.

(a) Write down the likelihood of this sample.

(b) Derive the maximum likelihood estimate of λ with the given sample.

(c) Suppose instead the sample is from the same Poisson distribution but truncated from below at zero; that is, $X_i \in (1, 2, 3, ...)$. Write down the likelihood of this truncated sample. (Note: $k! = 1$ when $k = 0$).

4. (20 points) Consider the following data-generating process:

$$Y_t = \epsilon_t + \theta \epsilon_{t-1},$$

where ϵ_t is a white noise process with mean zero and variance σ^2. Assume $\theta \neq 0$.

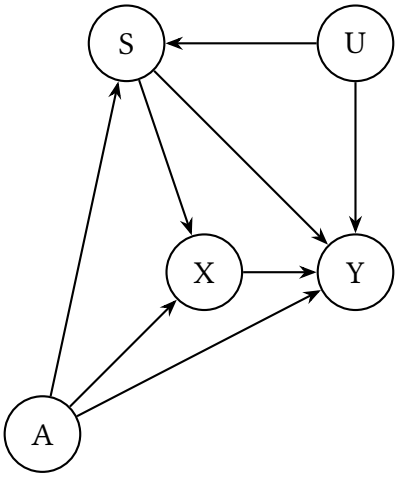
(a) Show whether Y_t is covariance stationary. Provide a formal justification based on the definition of covariance stationarity.

(b) Derive the autocovariance functions of Y_t for $h = 0, 1, 2$.

(c) Suppose a researcher mistakenly regresses Y_t on Y_{t-1} using a simple OLS model. What issues could arise from this misspecification? Briefly explain.

(d) If θ were very close to -1, how would the behavior of Y_t differ from a typical white noise process? Provide intuition.

5. (15 points) Consider a research project investigating the total causal effect of HIV status (X) on the risk of stroke (Y). The study also collects information on smoking behavior (S) and age (A). Unobserved background characteristics, such as genetic predisposition and family history, are summarized by the unobservable variable (U). The assumed data-generating process is represented by the following Directed Acyclic Graph (DAG):



(a) Identify all backdoor paths between X and Y. For each path, indicate whether it is open or blocked by default, and specify which variable(s) must be controlled to block the path.

(b) Propose a minimal sufficient adjustment set that satisfies the backdoor criterion for identifying the total causal effect of X on Y. Write the corresponding regression specification and briefly justify your selection.

(c) Suppose a researcher includes S in the regression model specified in part (b). Can the estimated coefficient on S be interpreted as the direct causal effect of smoking behavior on the risk of stroke? Justify your answer based on the structure of the DAG.

(d) Assume that Y is a binary variable. Suggest an appropriate econometric model that can be used to estimate the causal effect identified in part (b).

6. (20 pts) Consider the following stage game:

		Player 2 L	Player 2 M	Player 2 H
Player 1	L	$(10, 10)$	$(3, 15)$	$(0, 7)$
	M	$(15, 3)$	$(7, 7)$	$(-4, 5)$
	H	$(7, 0)$	$(5, -4)$	$(-15, -15)$

(a) Find all pure strategy NE of this game. Then, restricting attention to pure strategies only, compute each player's min-max payoff and identify the pure strategy profile that implements it.

(b) Determine the set of discount factors δ for which (L, L) be supported as an SPNE outcome in every period using the grim trigger strategy: If any deviation is observed, the players revert to playing the stage-game NE forever.

(c) Consider the following **one-period punishment strategy**:

- If any player deviates from (L, L), the players play (H, H) in the following period.
- From the next period onward, they return to (L, L).
- Any deviation during a punishment phase extends the punishment by one additional period.

For what values of δ can this strategy be sustained as a SPNE? Hint: Recall that SPNE only requires that there is no profitable one-shot deviation after any history.

(d) Now consider a **targeted punishment strategy**:

- A unilateral deviation from (L, L) triggers the min-max punishment of the deviator forever.
- If a punishing player deviates during the min-max punishment phase, they become the target of their own min-max punishment from that point onward.
- If both players deviate simultaneously, the game reverts to (L, L) in the next period.

Determine the values of δ for which this strategy can be supported as a SPNE.